What you'll be able to do!

machine translation "hello!" → "bonjour!"

document search "Can I get a refund?" →

"What's your return policy?"

...

"May I get my money back?"

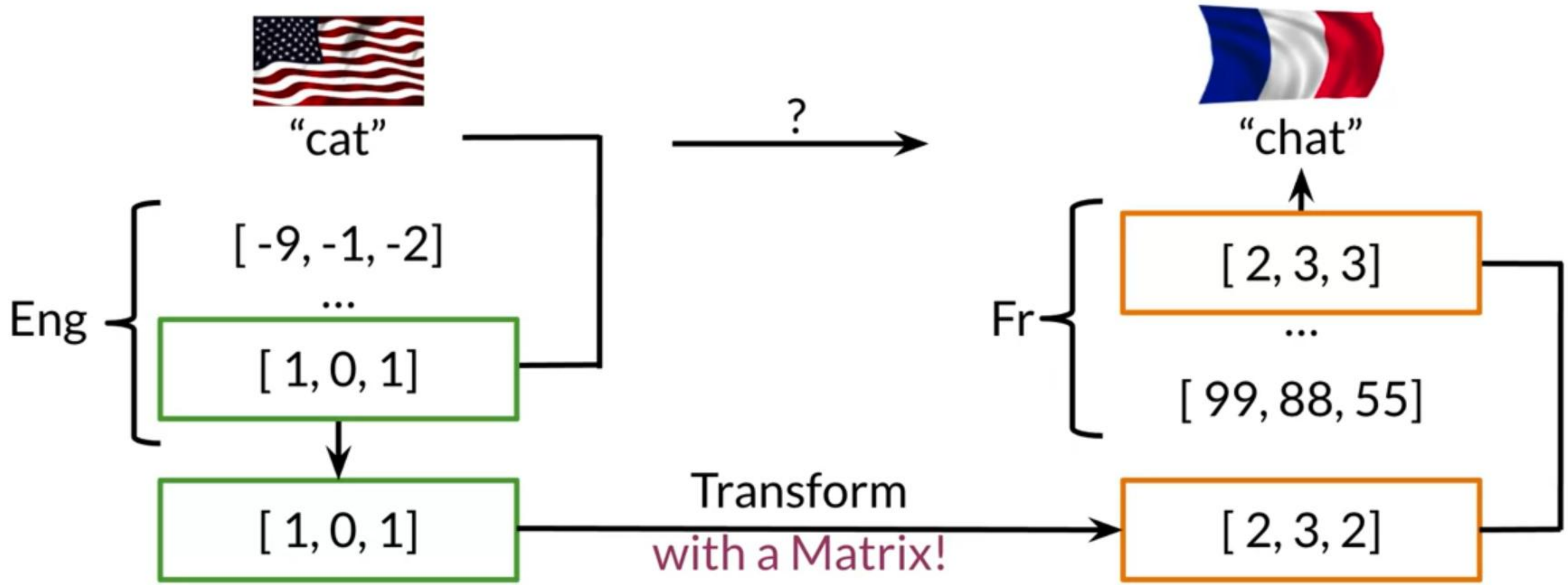
Learning Objectives

- Transform vector
- “K nearest neighbors”
- Hash tables
- Divide vector space into regions
- Locality sensitive hashing
- Approximated nearest neighbors

Machine translation

Document search

Overview of Translation



Transforming vectors

```
R = np.array([[2,0],
              [0,-2]])

x = np.array([[1,1]])

np.dot(x,R)
```

```
array([[2,-2]])
```

Align word vectors

$$\mathbf{XR} \approx \mathbf{Y}$$

$$\underbrace{\begin{pmatrix} [\text{“cat” vector}] \\ [\text{... vector}] \\ [\text{“zebra” vector}] \end{pmatrix}}_{\mathbf{X}} \qquad \underbrace{\begin{pmatrix} [\text{“chat” vecteur}] \\ [\text{... vecteur}] \\ [\text{“zébresse” vecteur}] \end{pmatrix}}_{\mathbf{Y}}$$

subsets of the full vocabulary

Solving for R

initialize R

in a loop:

$$Loss = \| \mathbf{XR} - \mathbf{Y} \|_F$$

$$g = \frac{d}{dR} Loss$$ gradient

$$R = R - \alpha g$$ update

Frobenius norm

Frobenius norm

```
A = np.array([[2,2],
              [2,2]])

A_squared = np.square(A)
A_squared
```

```
array([[4,4],
       [4,4]])
```

```
A_Frobenious = np.sqrt(np.sum(A_squared))
A_Frobenious
```

```
4.0
```

Frobenius norm squared

$$\|\mathbf{XR} - \mathbf{Y}\|_F^2$$

$$\mathbf{A} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

$$\|\mathbf{A}\|_F^2 = \left(\sqrt{2^2 + 2^2 + 2^2 + 2^2}\right)^2$$

$$\|\mathbf{A}\|_F^2 = 16$$

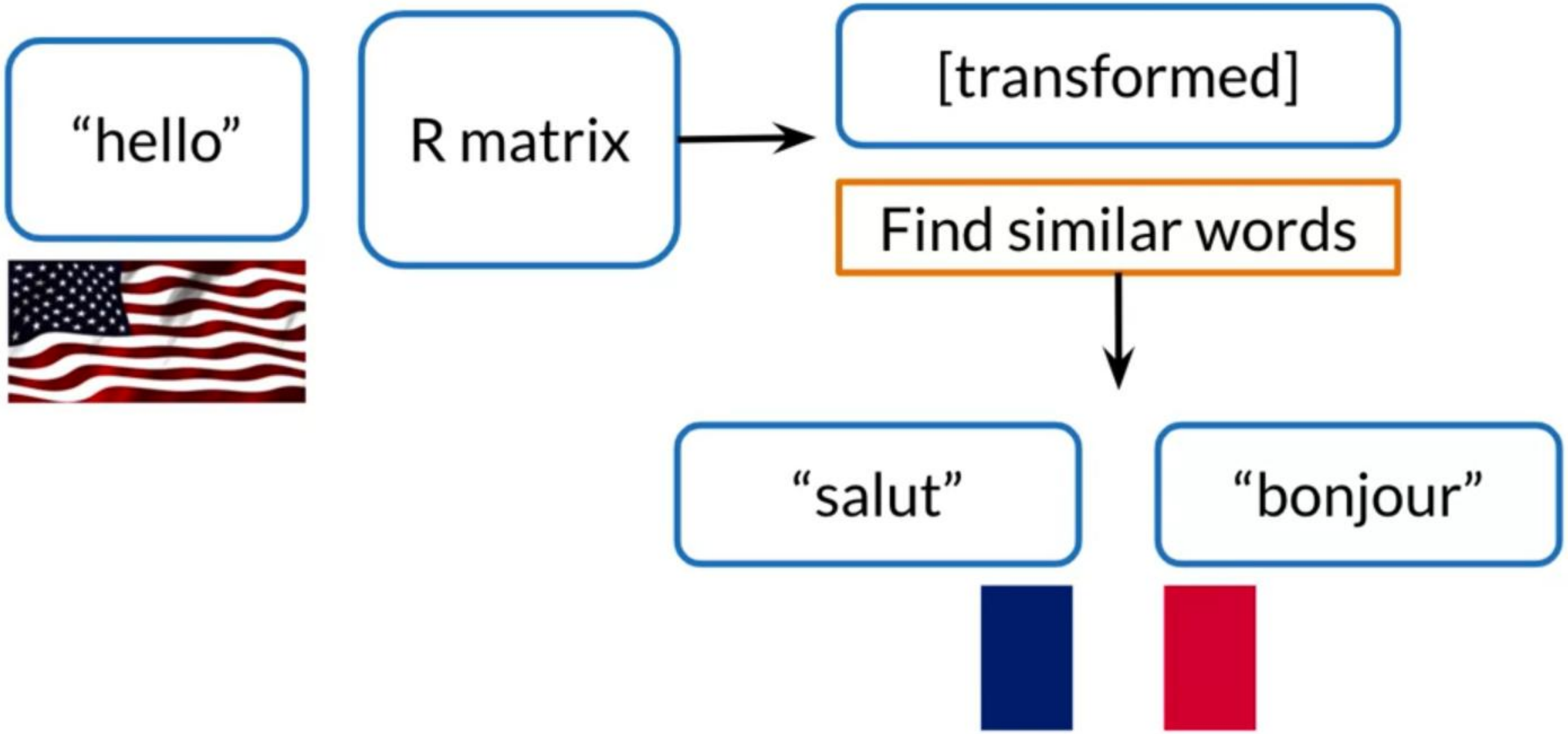
Gradient

$$Loss = \|\mathbf{XR} - \mathbf{Y}\|_F^2$$

$$g = \frac{d}{dR}Loss = \frac{2}{m}\left(\mathbf{X}^T(\mathbf{XR} - \mathbf{Y})\right)$$

$\boxed{\text{Implement in the assignment!}}$

Finding the translation



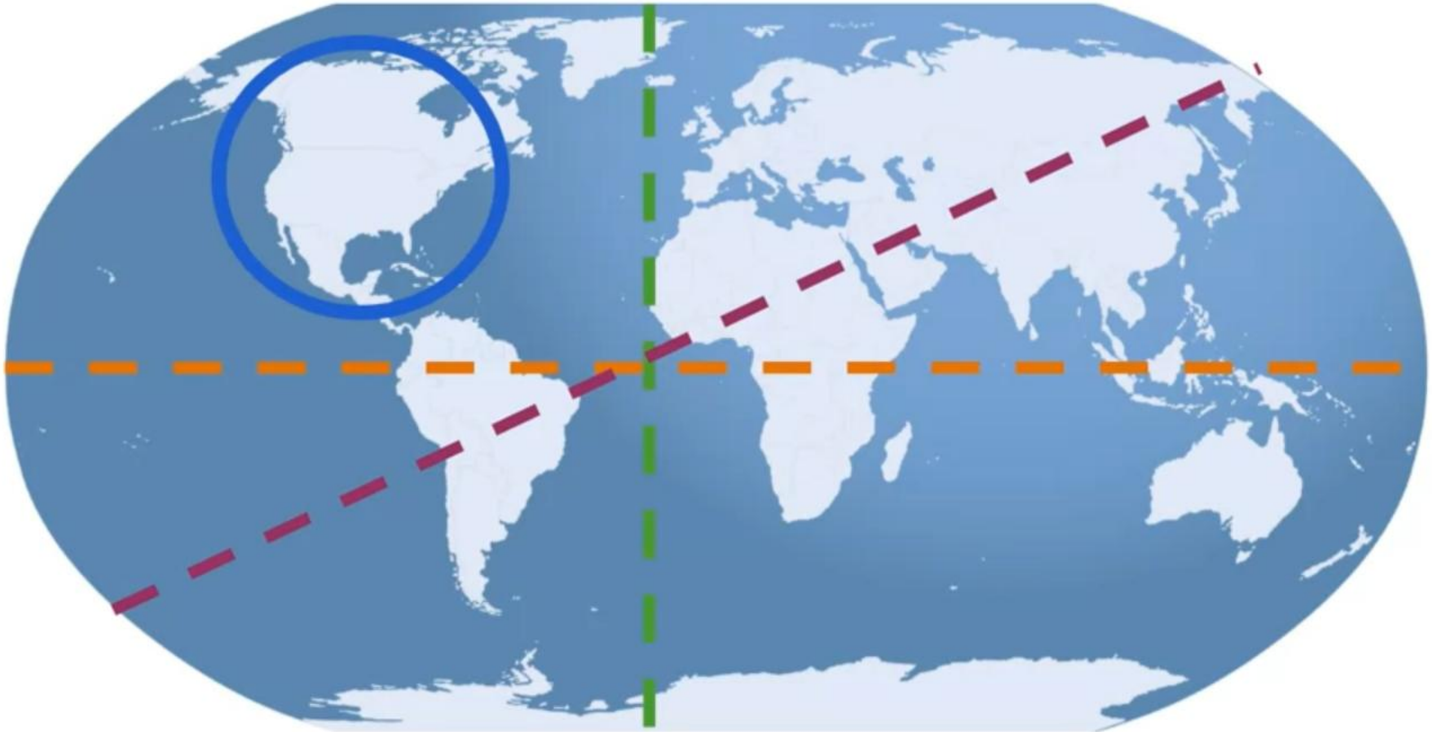
Nearest neighbours

You

San Francisco

Friend	Location	Nearest
	Shanghai	2
	Bangalore	3
	Los Angeles	1

Nearest neighbors

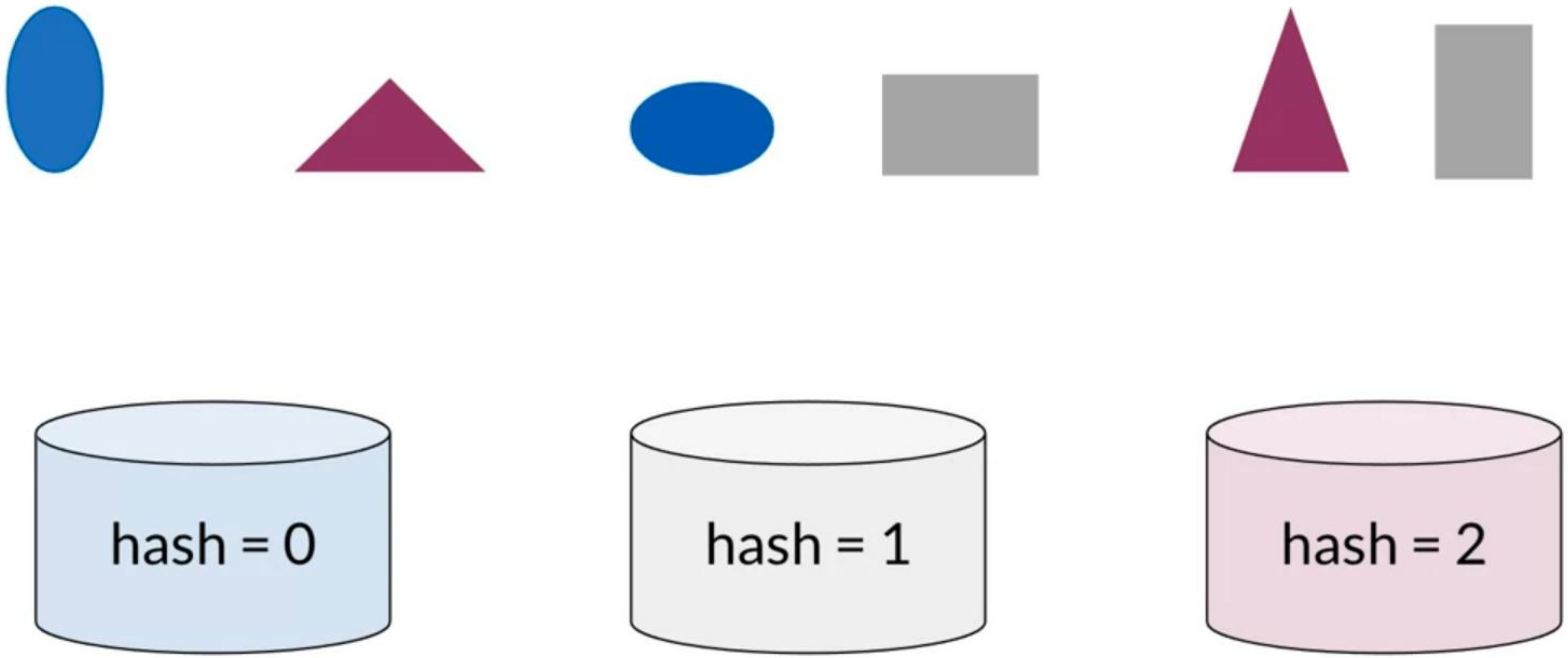


Hash
tables!

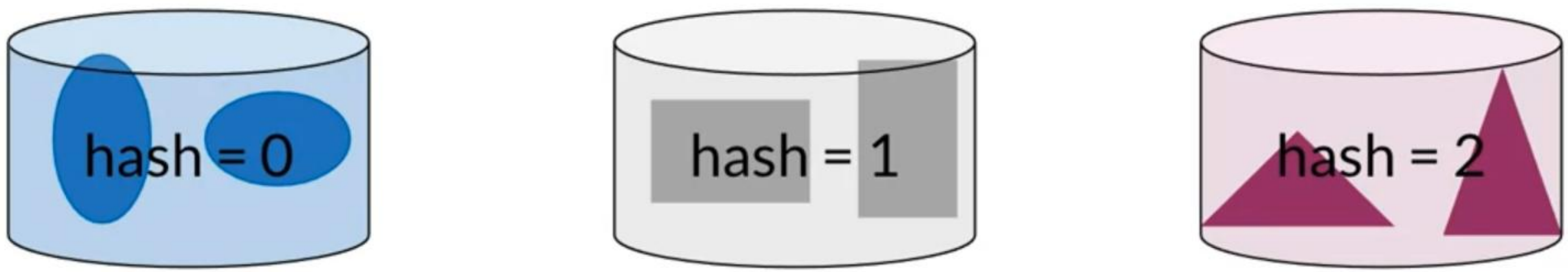
Summary

- K-nearest neighbors, for closest matches
- Hash tables

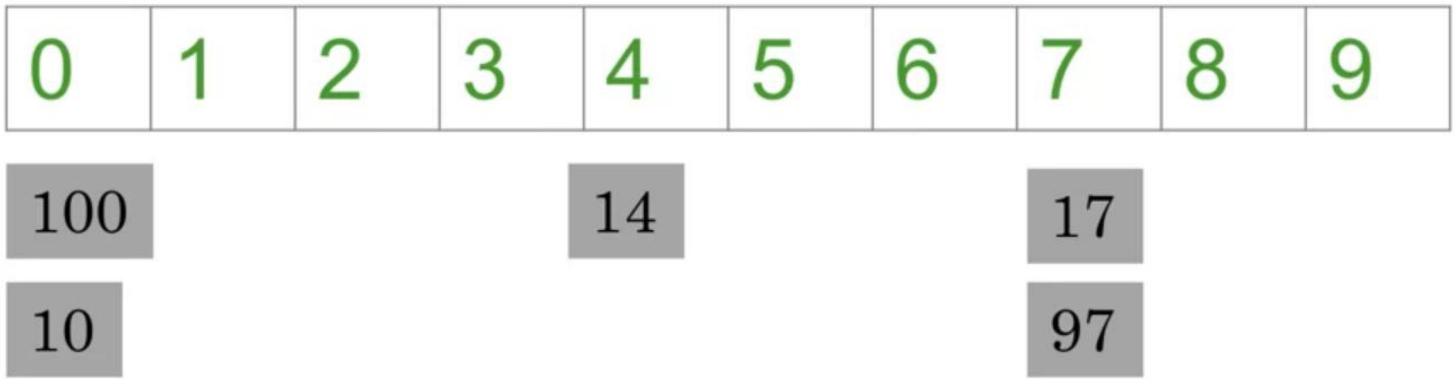
Hash tables



Hash tables



Hash function



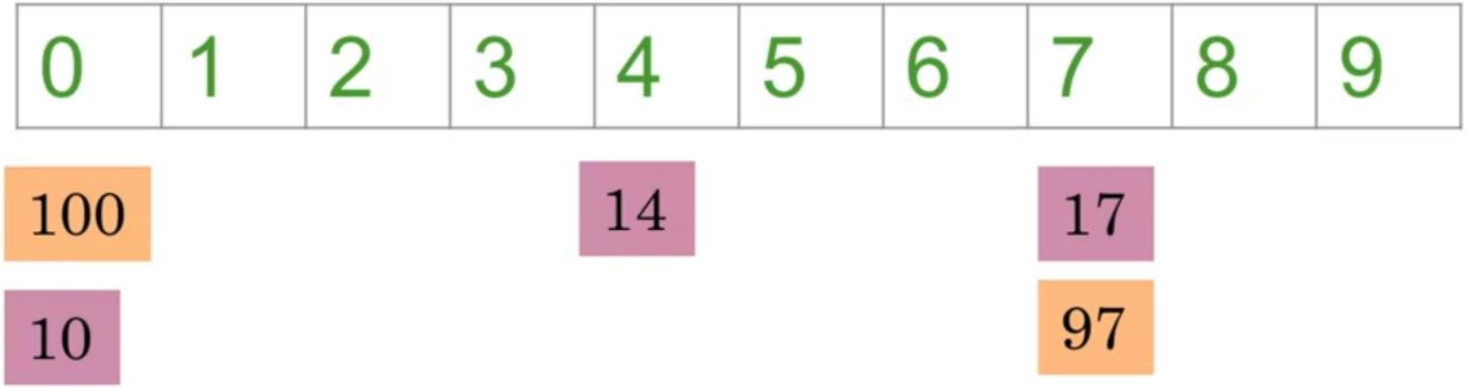
Hash function (vector) $\longrightarrow$ Hash value

Hash value = vector % number of buckets

Create a basic hash table

```
def basic_hash_table(value_l,n_buckets):

    def hash_function(value_l,n_buckets):
        return int(value) % n_buckets

    hash_table = {i:[] for i in range(n_buckets)}
    for value in value_l:
        hash_value = hash_function(value,n_buckets)
        hash_table[hash_value].append(value)

    return hash_table
```

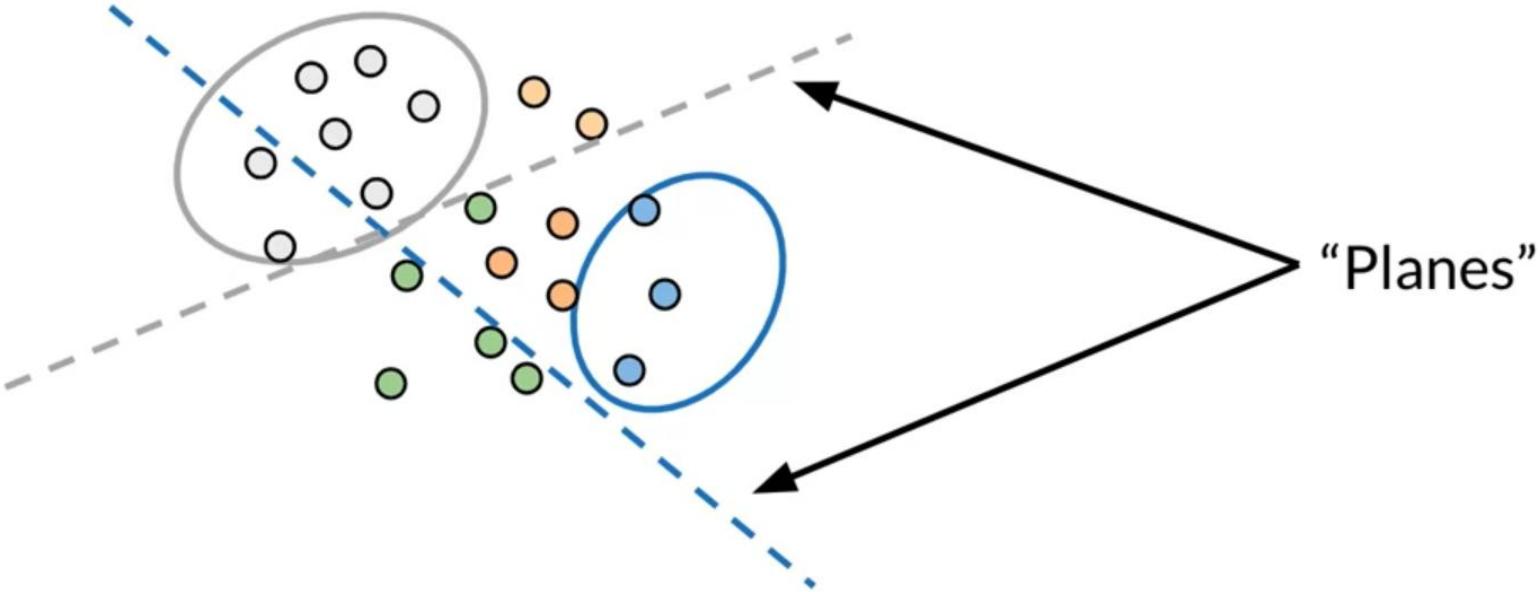
Hash function



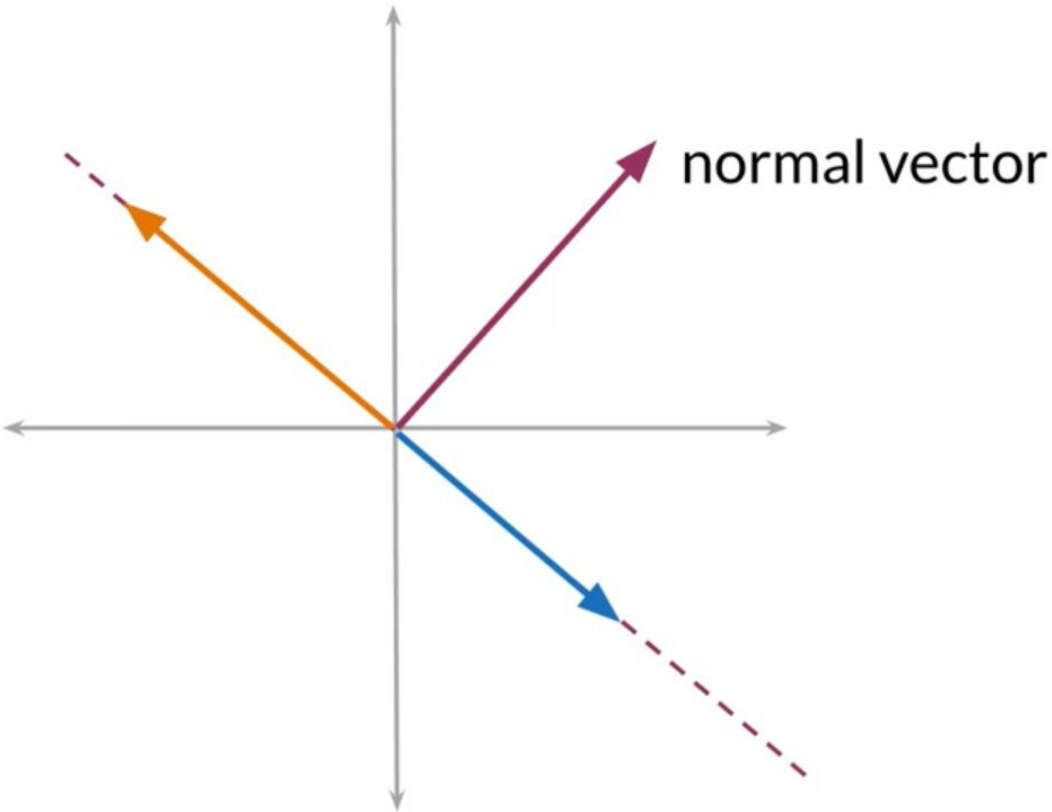
Hash function by location?

Locality sensitive hashing, next!

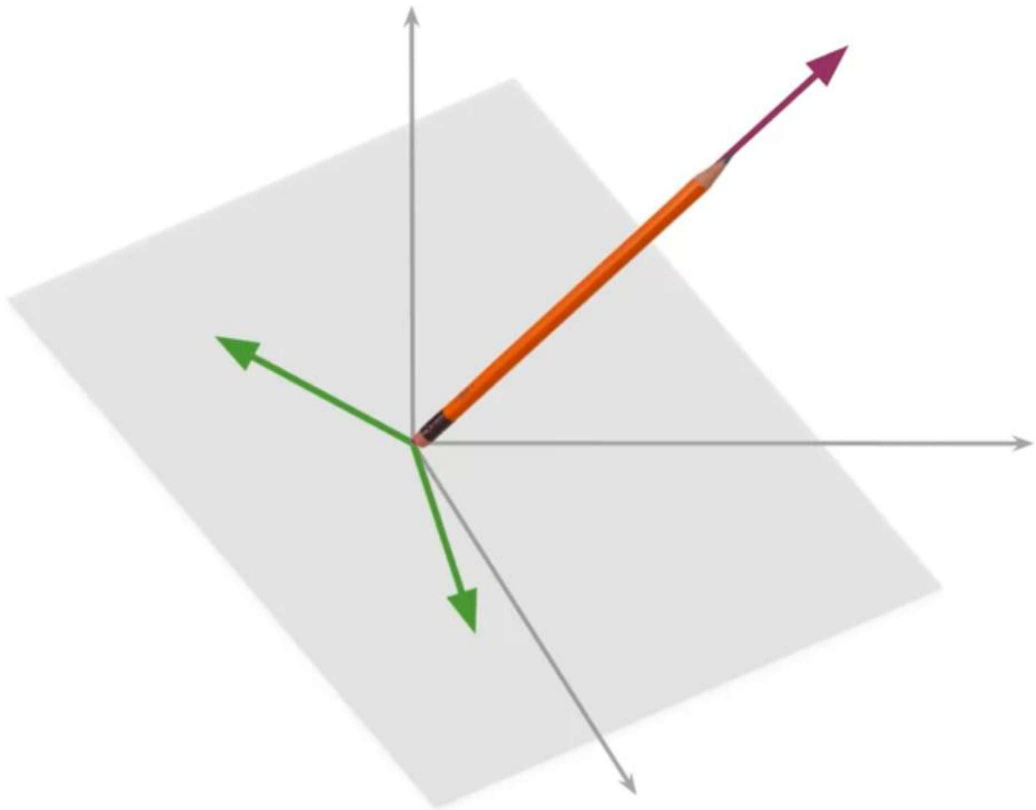
Locality Sensitive Hashing



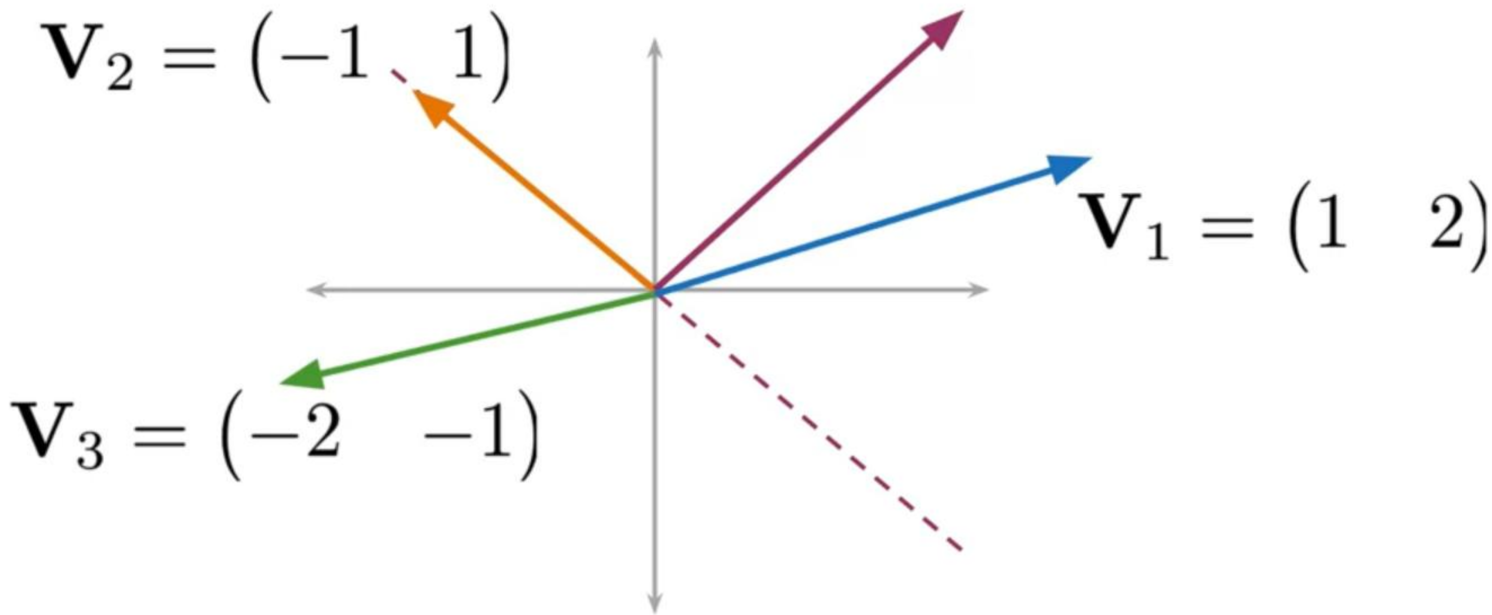
Planes



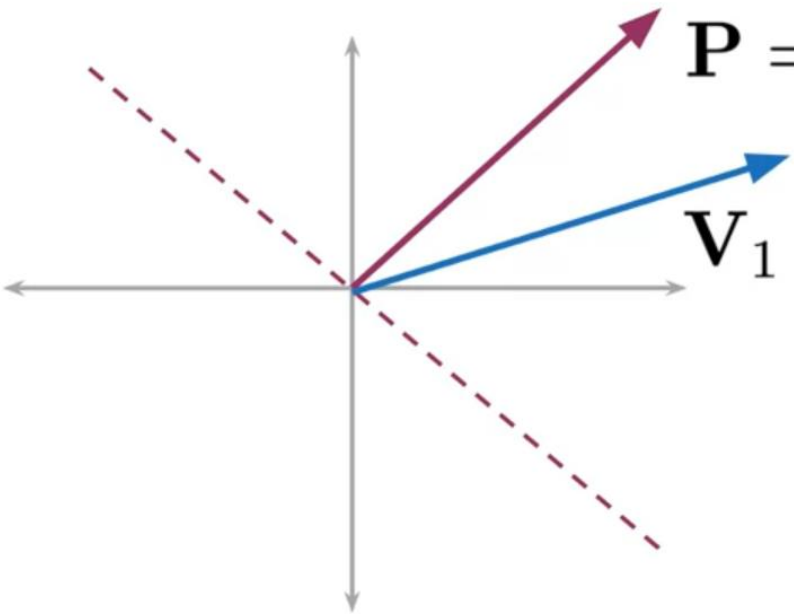
Planes



Which side of the plane?



Which side of the plane?



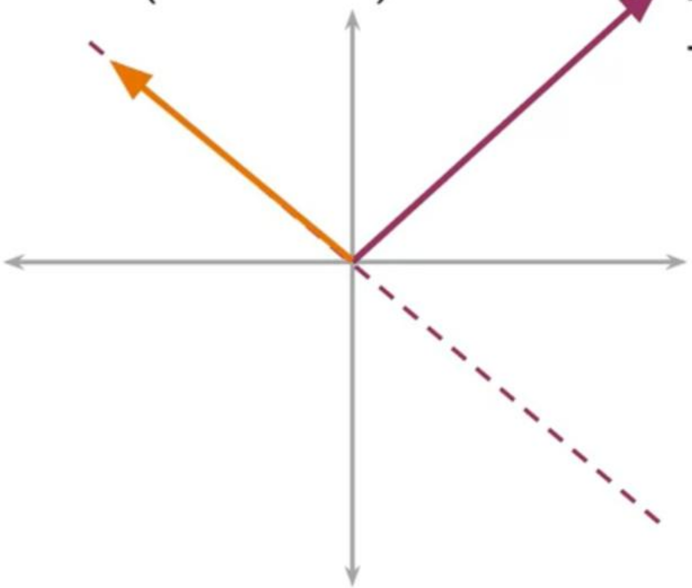
$$\mathbf{P} = \begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$\mathbf{V}_1 = \begin{pmatrix} 1 & 2 \end{pmatrix}$$

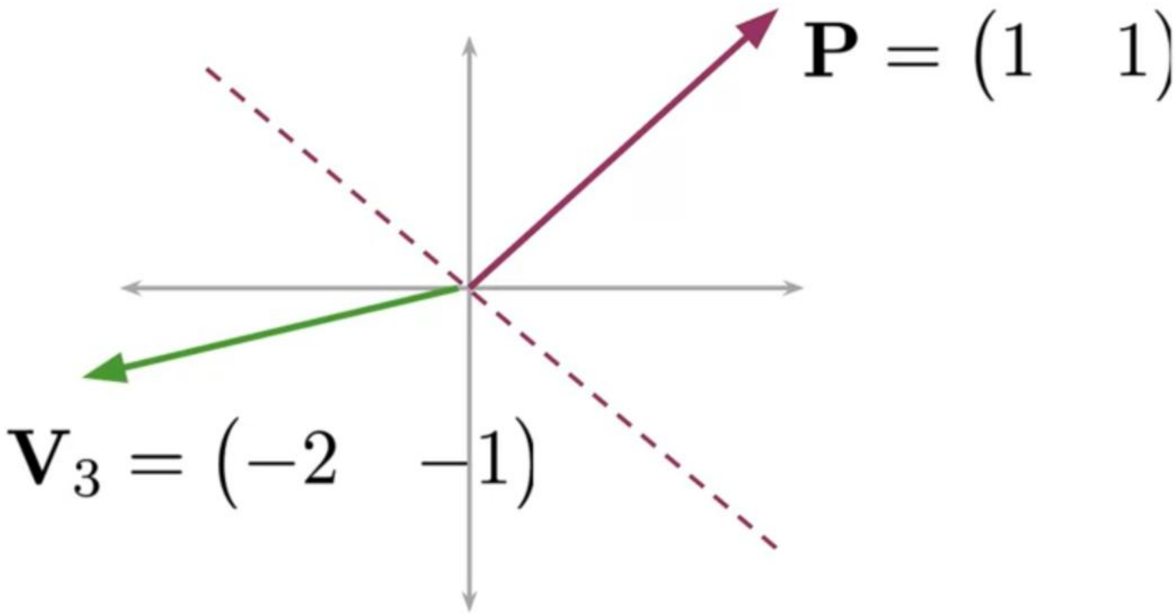
Which side of the plane?

$\mathbf{V}_2 = \begin{pmatrix} -1 & 1 \end{pmatrix}$

$\mathbf{P} = \begin{pmatrix} 1 & 1 \end{pmatrix}$

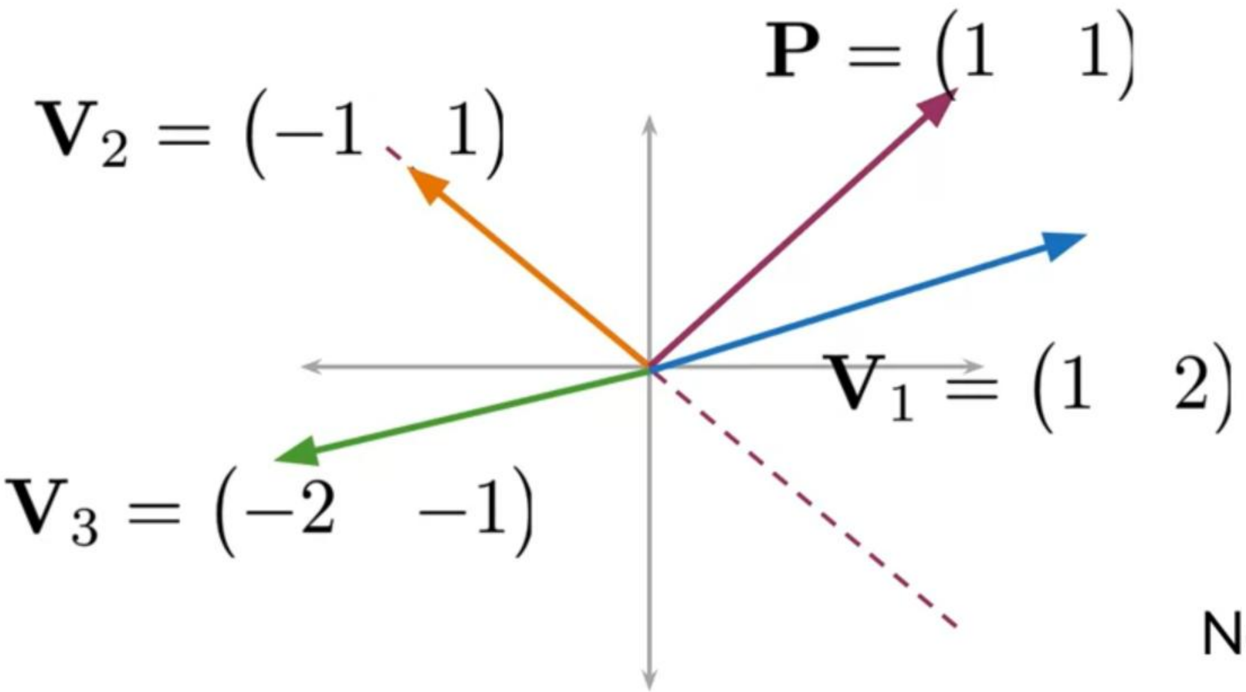


Which side of the plane?



$$\mathbf{P}\mathbf{V}_3^T = -3$$

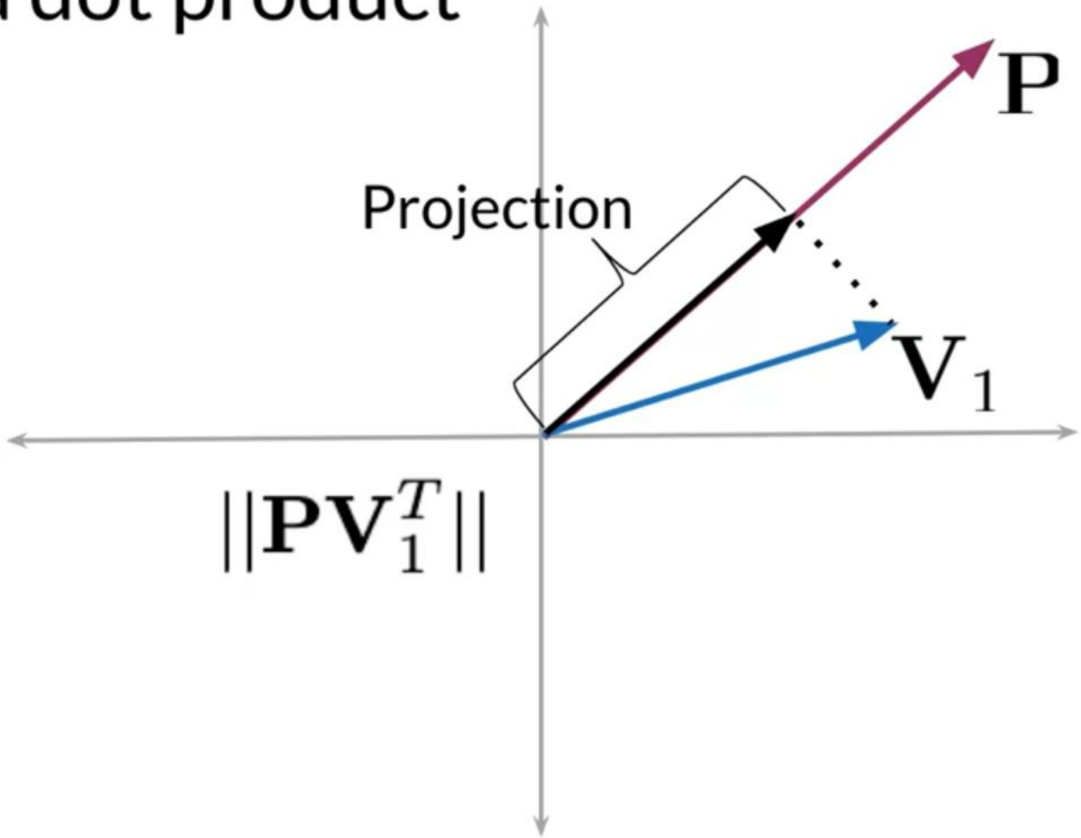
Which side of the plane?



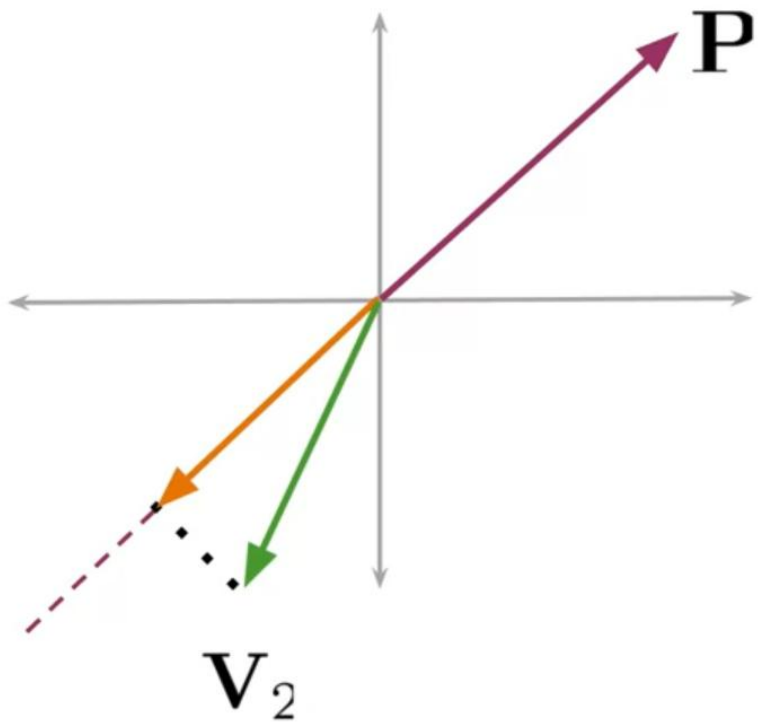
$$\mathbf{P}\mathbf{V}_1^T = 3$$
$$\mathbf{P}\mathbf{V}_2^T = 0$$
$$\mathbf{P}\mathbf{V}_3^T = -3$$

Notice the signs?

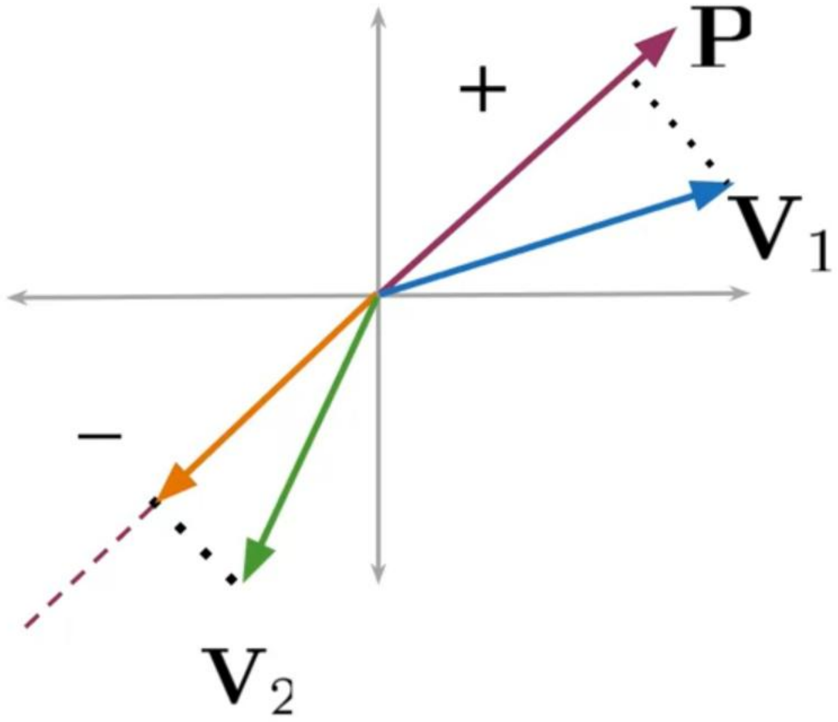
Visualizing a dot product



Visualizing a dot product



Visualizing a dot product



Sign indicates direction

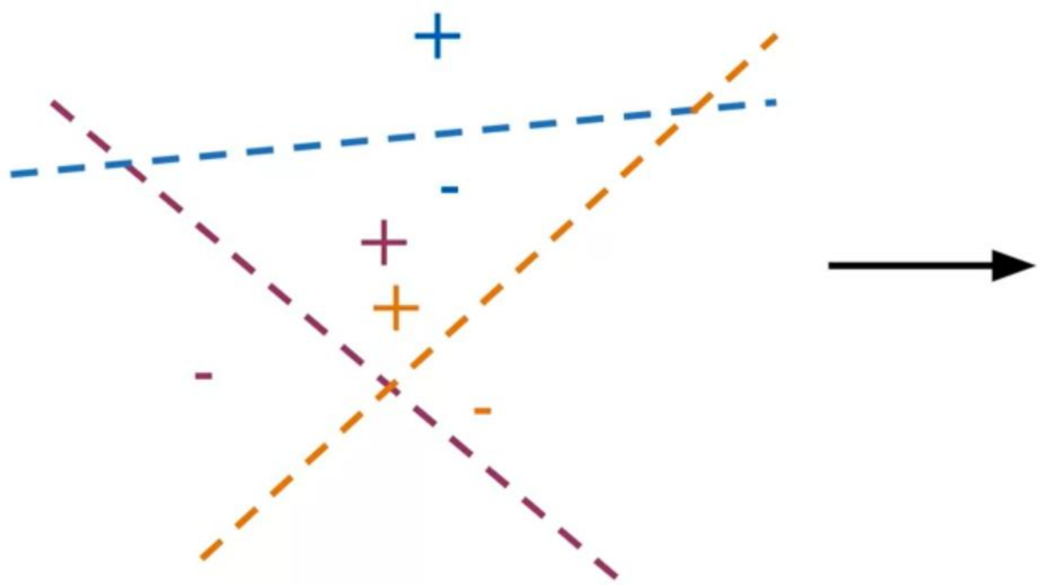
Which side of the plane?

```
def side_of_plane(P,v):
    dotproduct = np.dot(P,v.T)
    sign_of_dot_product = np.sign(dotproduct)
    sign_of_dot_product_scalar= np.asscalar(sign_of_dot_product)
    return sign_of_dot_product_scalar
```

Outline

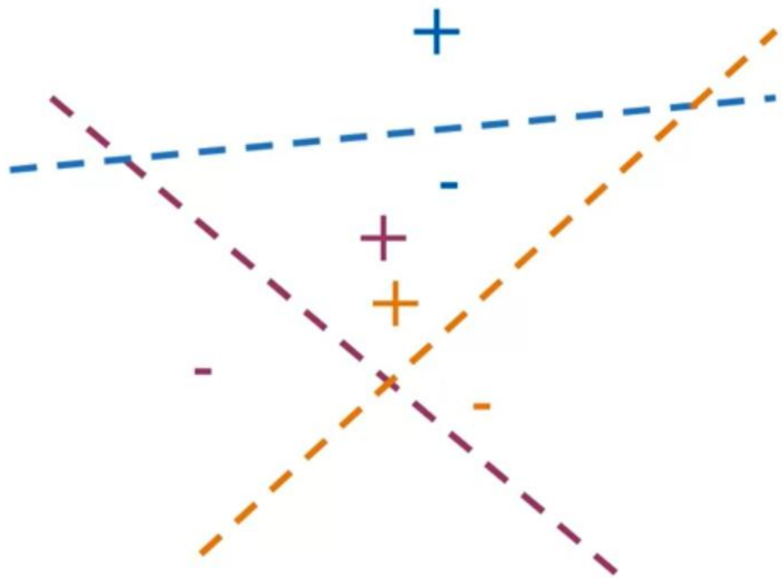
- Multiple planes $\longrightarrow$ Dot products $\longrightarrow$ Hash values

Multiple planes



single hash value?

Multiple planes, single hash value?



$$\boxed{\mathbf{P}_1\mathbf{v}^T = 3, sign_1 = +1, h_1 = 1}$$

$$\boxed{\mathbf{P}_2\mathbf{v}^T = 5, sign_2 = +1, h_2 = 1}$$

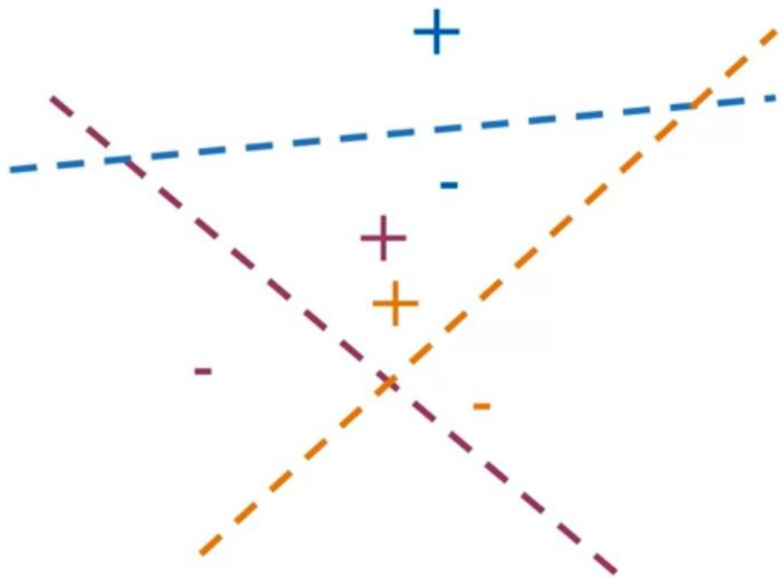
$$\boxed{\mathbf{P}_3\mathbf{v}^T = -2, sign_3 = -1, h_3 = 0}$$

$$hash = 2^0 \times h_1 + 2^1 \times h_2 + 2^2 \times h_3$$

$$= 1 \times 1 + 2 \times 1 + 4 \times 0$$

$$= 3$$

Multiple planes, single hash value!



$$sign_i \geq 0, \rightarrow h_i = 1$$

$$sign_i < 0, \rightarrow h_i = 0$$

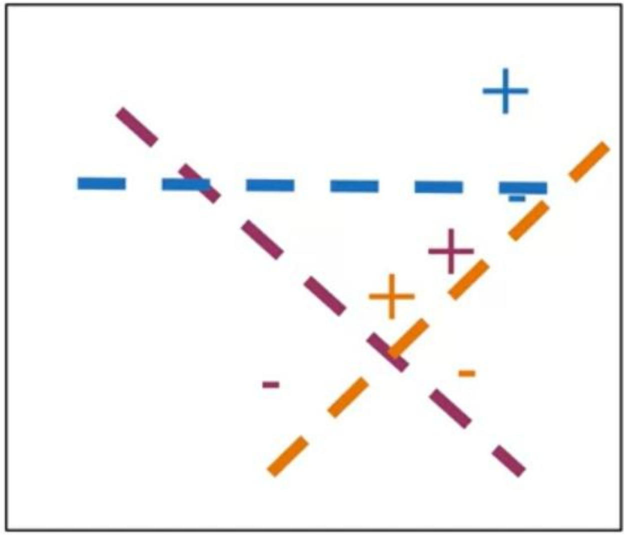
$$\text{hash} = \sum_{i}^{H} 2^i \times h_i$$

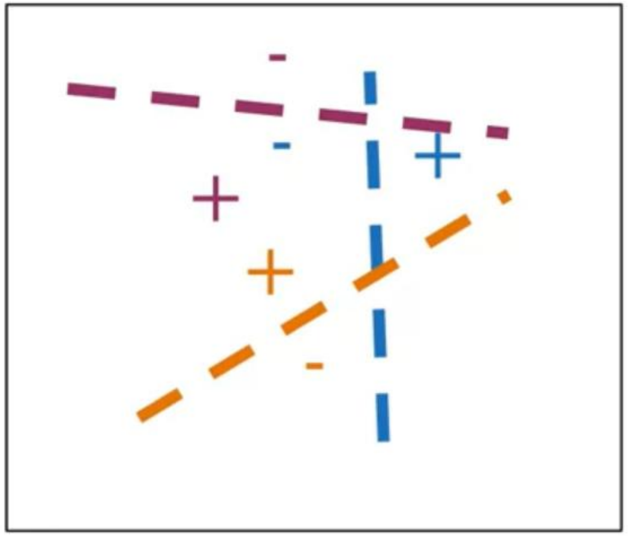
Multiple planes, single hash value!!

```python
def hash_multiple_plane(P_l,v):

    hash_value = 0

    for i, P in enumerate(P_l):
        sign = side_of_plane(P,v)
        hash_i = 1 if sign >=0 else 0
        hash_value += 2**i * hash_i

    return hash_value
```

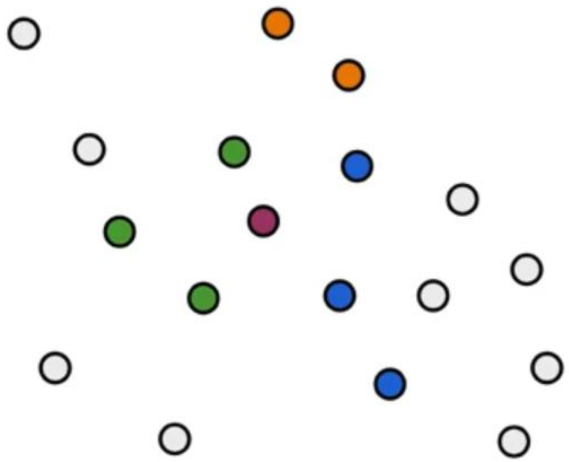
Try it!

Random planes





Multiple sets of random planes



Approximate nearest
(friendly) neighbors

Make one set of random planes

```
num_dimensions = 2 #300 in assignment
num_planes = 3 #10 in assignment

random_planes_matrix = np.random.normal(
                       size=(num_planes,
                             num_dimensions))
```

```
array([[ 1.76405235  0.40015721]
       [ 0.97873798  2.2408932 ]
       [ 1.86755799 -0.97727788]])
```

```
v = np.array([[2,2]])
```

```
def side_of_plane_matrix(P,v):
     dotproduct = np.dot(P,v.T)
     sign_of_dot_product = np.sign(dotproduct)
     return sign_of_dot_product

num_planes_matrix = side_of_plane_matrix(
                    random_planes_matrix,v)
```

```
array([[1.]
       [1.]
       [1.])
```

See notebook for calculating the hash value!

Document representation

I love learning!	[?, ?, ?]
I	[1, 0, 1]
	+
love	[-1, 0, 1]
	+
learning	[1, 0, 1]
	=
I love learning!	[1, 0, 3]

Document Search

K-NN!

Document vectors

```
word_embedding = {"I": np.array([1,0,1]),
                  "love": np.array([-1,0,1]),
                  "learning": np.array([1,0,1])}

words_in_document = ['I', 'love', 'learning']

document_embedding = np.array([0,0,0])

for word in words_in_document:
    document_embedding += word_embedding.get(word,0)

print(document_embedding)
```

```
array([1 0 3])
```