Outline

- Vector space models
- Advantages
- Applications

Why learn vector space models?

Where are you heading?

Where are you from?

Different meaning

What is your age?

How old are you?

Same Meaning

Vector space models applications

- You eat *cereal* from a *bowl*
- You *buy* something and someone else *sells* it

Information Extraction

Machine Translation

Chatbots

Fundamental concept

"You shall know a word by the company it keeps" Firth, 1957

(Firth, J. R. 1957:11)

Summary

- Represent words and documents as vectors
- Representation that captures relative meaning

Outline

- Co-occurrence $\longrightarrow$ Vector representation
- Relationships between words/documents

Word by Word Design

Number of times they *occur together within a certain distance k*

I like simple data

I prefer simple raw data

k=2

	simple	raw	like	I
data	2	1	1	0

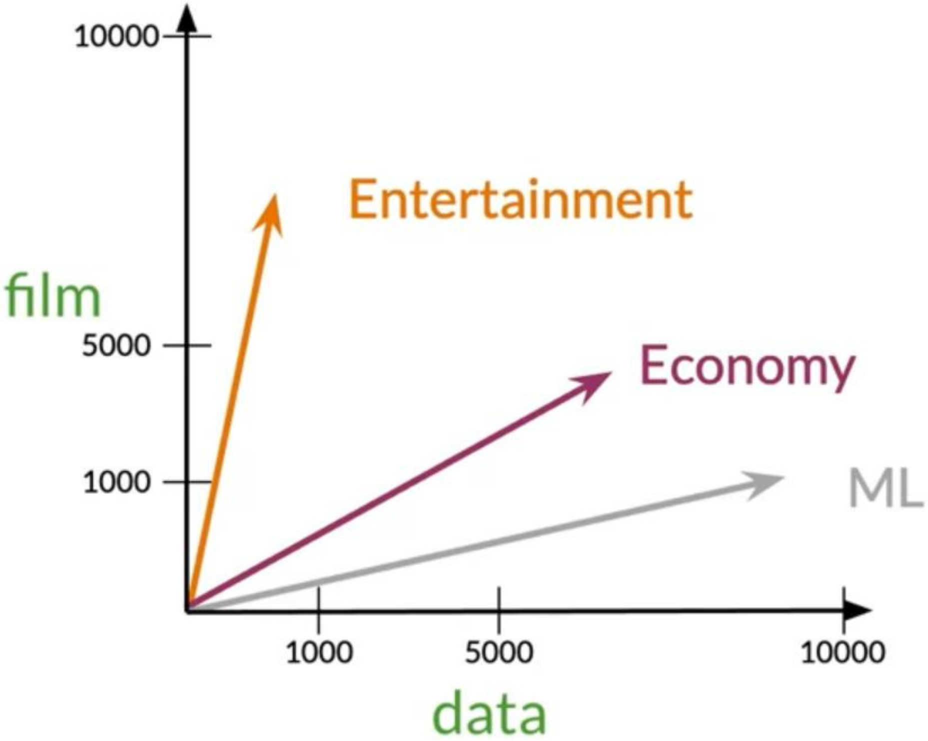
Word by Document Design

Number of times a word ***occurs within a certain category***

Entertainment	Economy	Machine Learning

	Entertainment	Economy	Machine Learning
data	500	6620	9320
film	7000	4000	1000

Vector Space



	Entertainment	Economy	ML
data	500	6620	9320
film	7000	4000	1000

Measures of "similarity:"
Angle
Distance

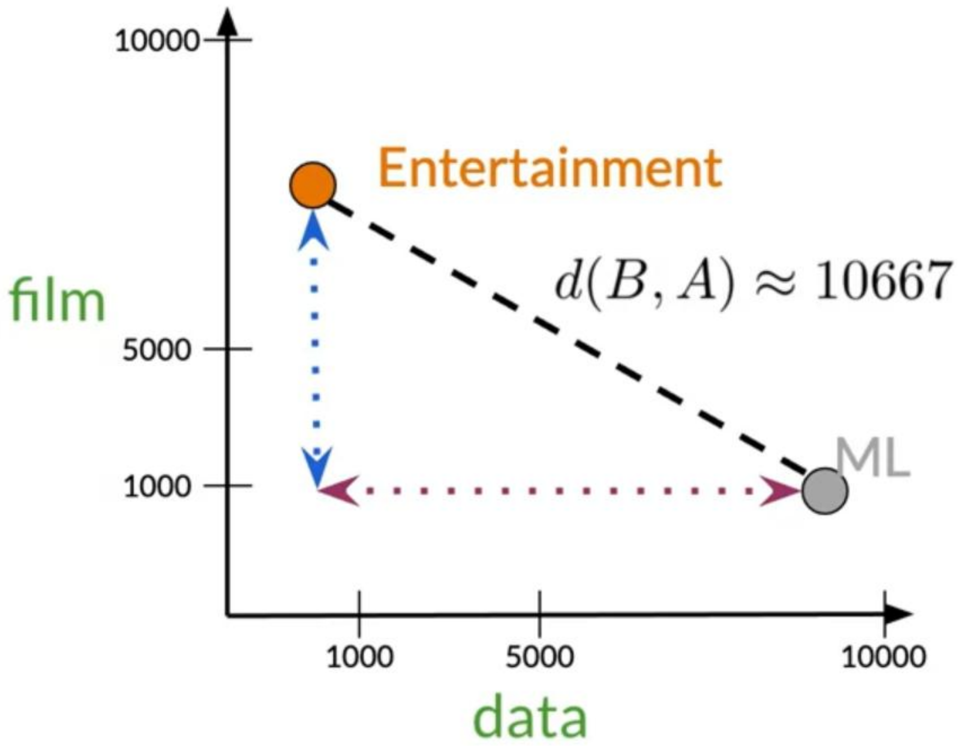
Summary

- W/W and W/D, *counts* of occurrence
- Vector Spaces $\longrightarrow$ Similarity between words/documents

Outline

- Euclidean distance
- N-dimension vector representations comparison

Euclidean distance



Corpus A: (500,7000)

Corpus B: (9320,1000)

$$d(B, A) = \sqrt{(B_1 - A_1)^2 + (B_2 - A_2)^2}$$

$$c^2 = a^2 + b^2$$

$$d(B, A) = \sqrt{(8820)^2 + (-6000)^2}$$

Euclidean distance for n-dimensional vectors

	data	boba ($\vec{w}$)	ice-cream ($\vec{v}$)
AI	6	0	1
drinks	0	4	6
food	0	6	8

$$= \sqrt{(1-0)^2 + (6-4)^2 + (8-6)^2}$$

$$= \sqrt{1+4+4} = \sqrt{9} = 3$$

$$d(\vec{v}, \vec{w}) = \sqrt{\sum_{i=1}^{n} (v_i - w_i)^2} \longrightarrow \text{Norm of } (\vec{v} - \vec{w})$$

Euclidean distance in Python

```python
# Create numpy vectors v and w
v = np.array([1, 6, 8])
w = np.array([0, 4, 6])

# Calculate the Euclidean distance d
d = np.linalg.norm(v-w)
# Print the result
print("The Euclidean distance between v and w is: ", d)
```

```
The Euclidean distance between v and w is: 3
```

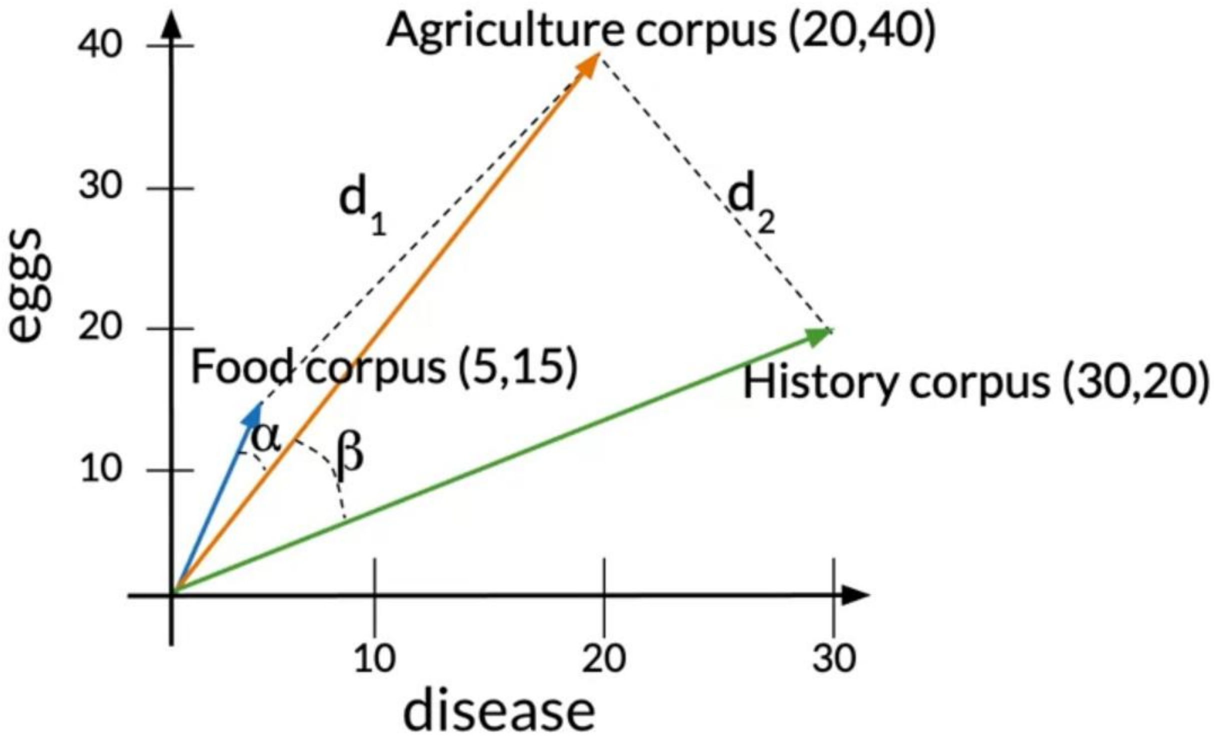
Summary

- Straight line between points
- Norm of the difference between vectors

Outline

- Problems with Euclidean Distance
- Cosine similarity

Euclidean distance vs Cosine similarity



Euclidean distance: $d_2 < d_1$

Angles comparison: $\beta > \alpha$

The cosine of the angle between the vectors

Summary

- Cosine similarity when corpora are different sizes

Outline

- How to get the cosine of the angle between two vectors
- Relation of this metric to similarity

Previous definitions

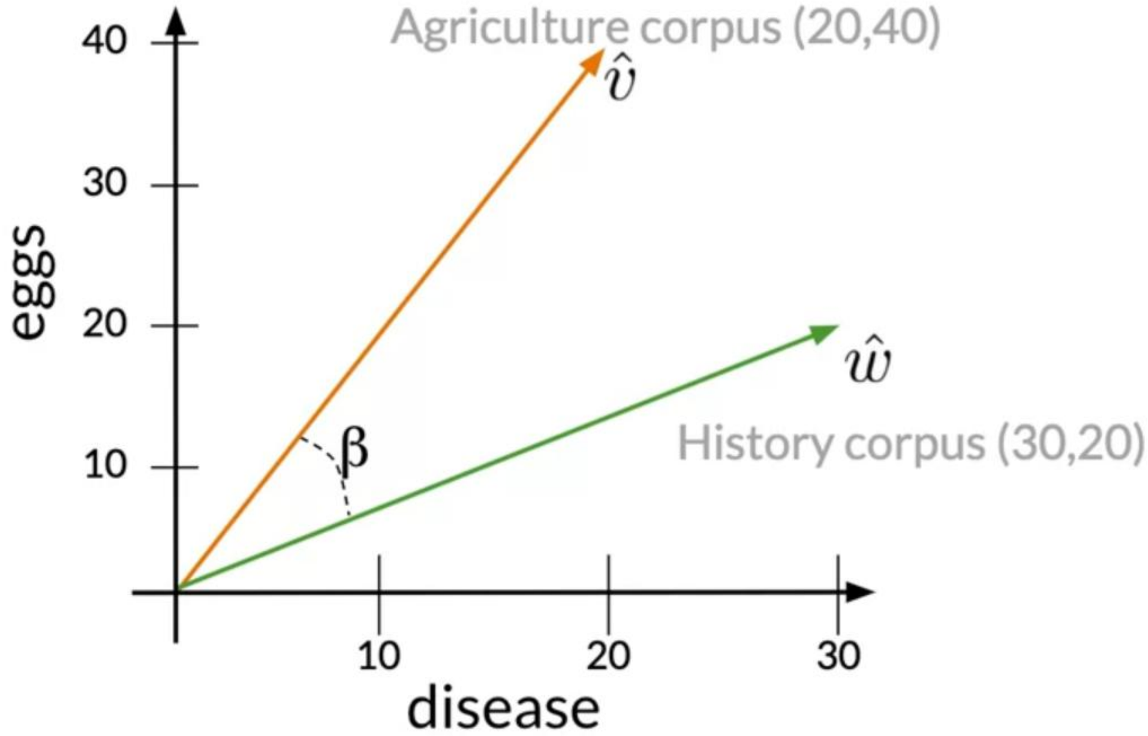
Vector norm

$$\|\vec{v}\| = \sqrt{\sum_{i=1}^{n} v_i^2}$$

Dot product

$$\vec{v}.\vec{w} = \sum_{i=1}^{n} v_i.w_i$$

Cosine Similarity



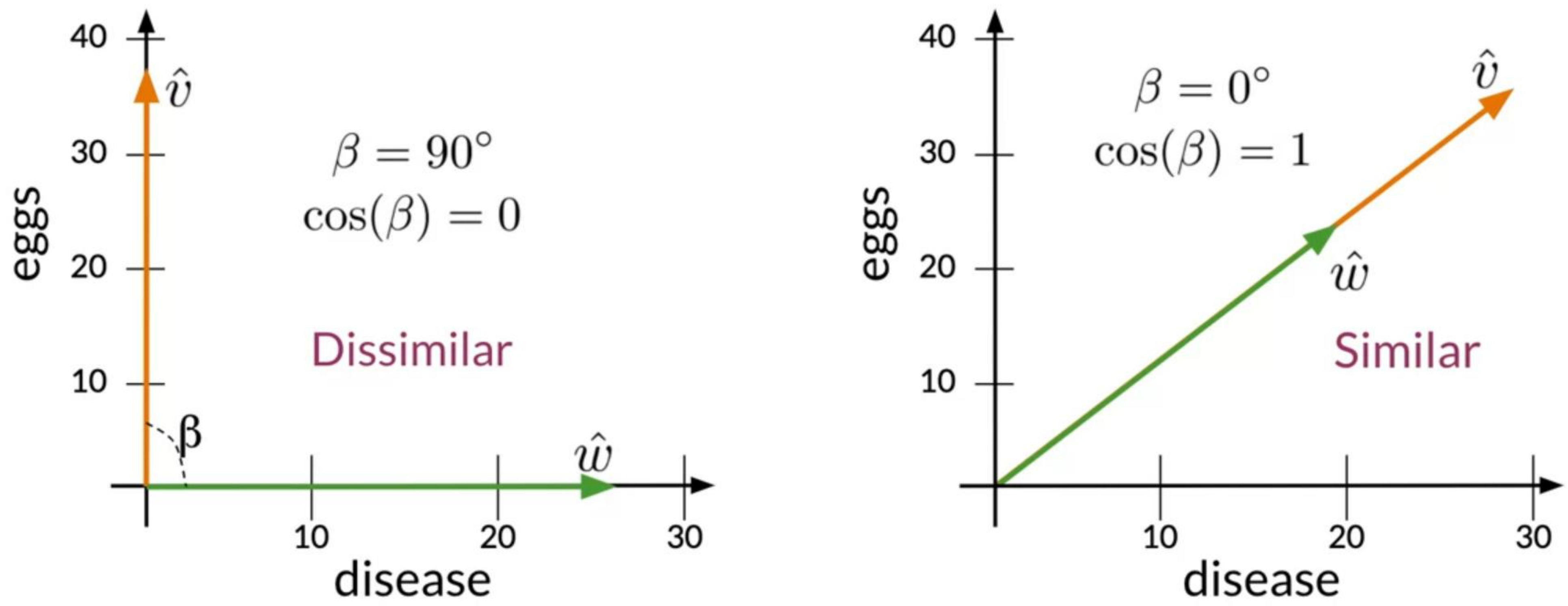
$$\hat{v} \cdot \hat{w} = \|\hat{v}\| \|\hat{w}\| \cos(\beta)$$

$$\boxed{\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{\|\hat{v}\| \|\hat{w}\|}}$$

$$= \frac{(20 \times 30) + (40 \times 20)}{\sqrt{20^2 + 40^2} \times \sqrt{30^2 + 20^2}}$$

$$= 0.87$$

Cosine Similarity



Summary

- Cosine $\propto$ Similarity
- Cosine Similarity gives values between 0 and 1

Outline

- How to use vector representations

Manipulating word vectors

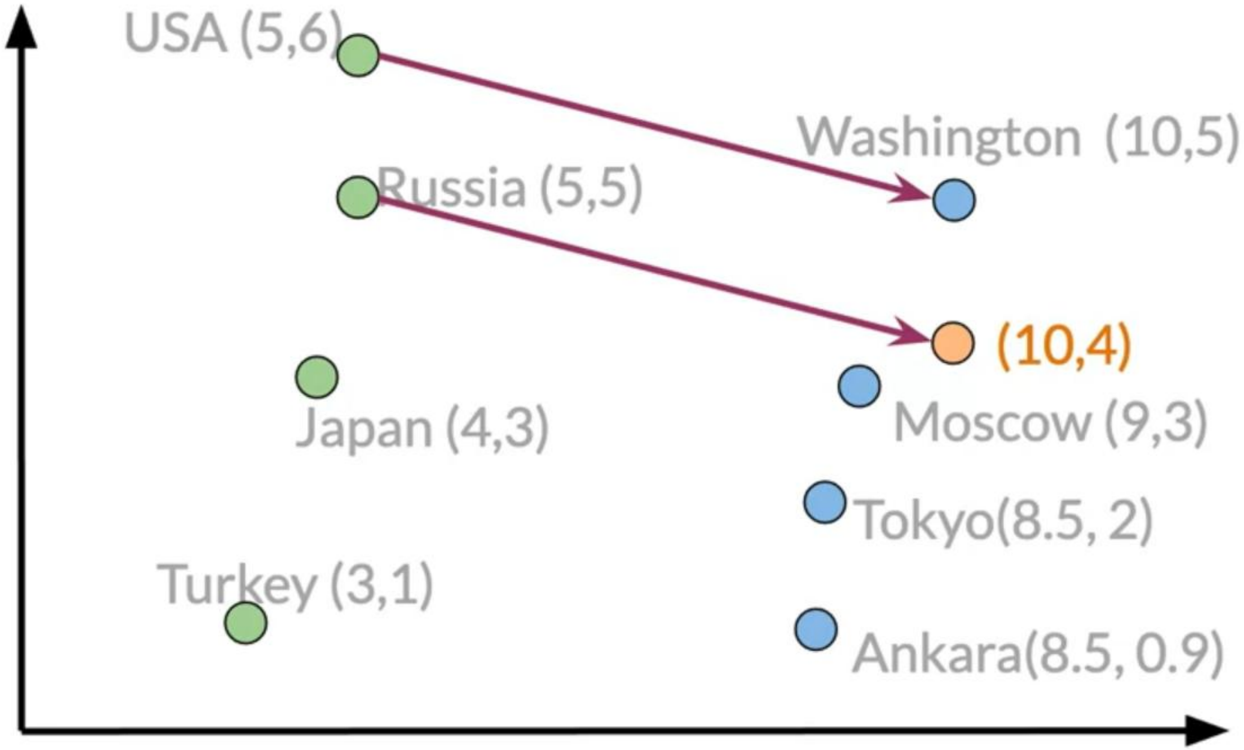
USA

Washington DC

Russia

?

Manipulating word vectors

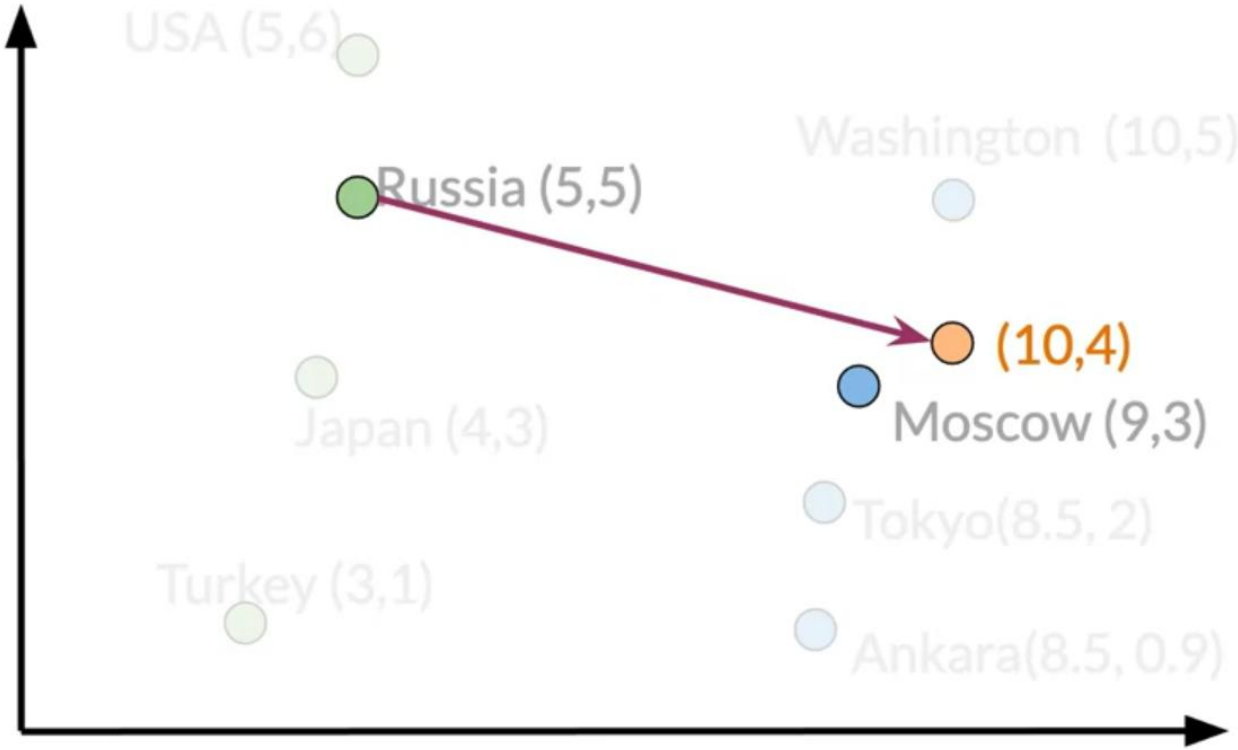


Washington - USA = $\begin{bmatrix} 5 & -1 \end{bmatrix}$

Russia + $\begin{bmatrix} 5 & -1 \end{bmatrix}$ = $\begin{bmatrix} 10 & 4 \end{bmatrix}$

[Mikolov et al, 2013, Distributed Representations of Words and Phrases and their Compositionality]

Manipulating word vectors



Washington - USA = $\begin{bmatrix} 5 & -1 \end{bmatrix}$

Russia + $\begin{bmatrix} 5 & -1 \end{bmatrix}$ = $\begin{bmatrix} 10 & 4 \end{bmatrix}$

Moscow

[Mikolov et al, 2013, Distributed Representations of Words and Phrases and their Compositionality]

Summary

- Use known relationships to make predictions

Outline

- Some motivation for visualization
- Principal Component Analysis

Visualization of word vectors

$\overbrace{\hspace{6em}}^{d > 2}$

oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

How can you visualize if your representation captures these relationships?

oil & gas

town & city

Visualization of word vectors

$\overbrace{\qquad\qquad}^{d > 2}$

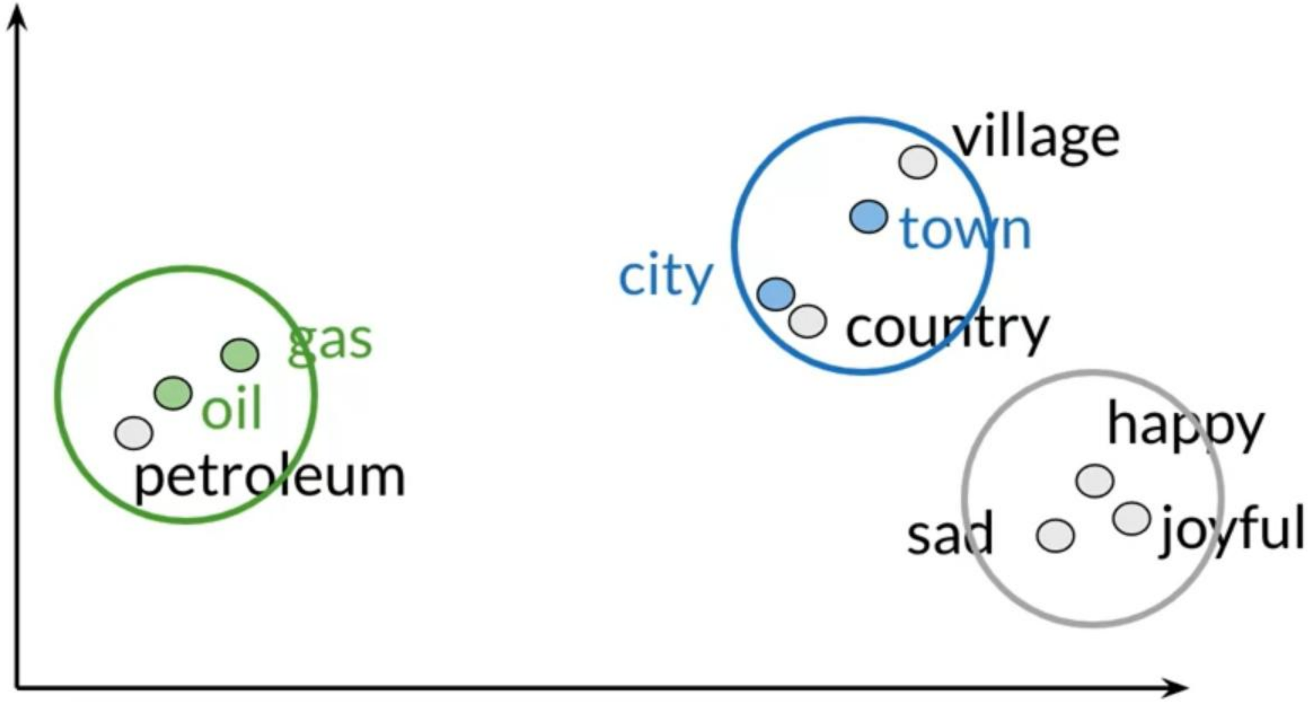
oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

$\xrightarrow{\text{PCA}}$

$\overbrace{\qquad\qquad}^{d = 2}$

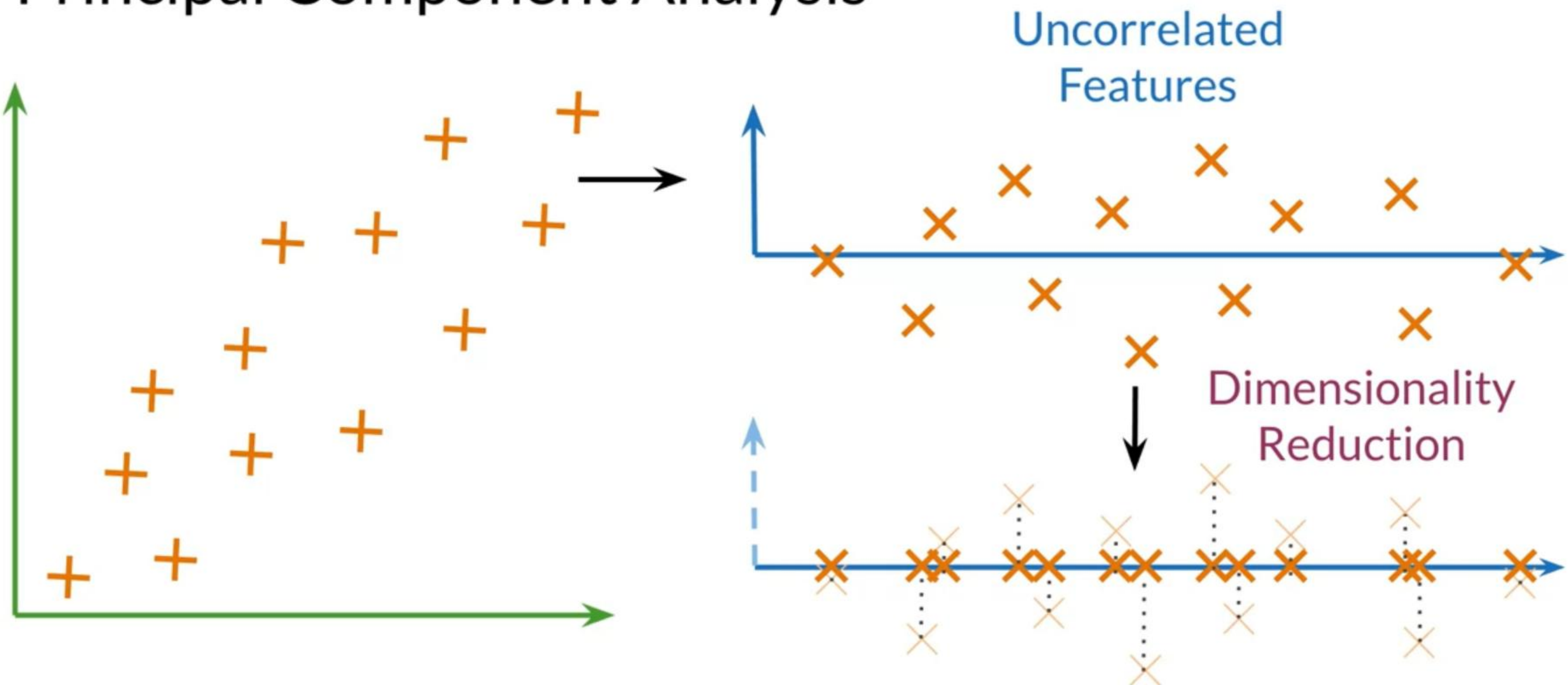
oil	2.30	21.2
gas	1.56	19.3
city	13.4	34.1
town	15.6	29.8

Visualization of word vectors



deeplearning.ai

Principal Component Analysis



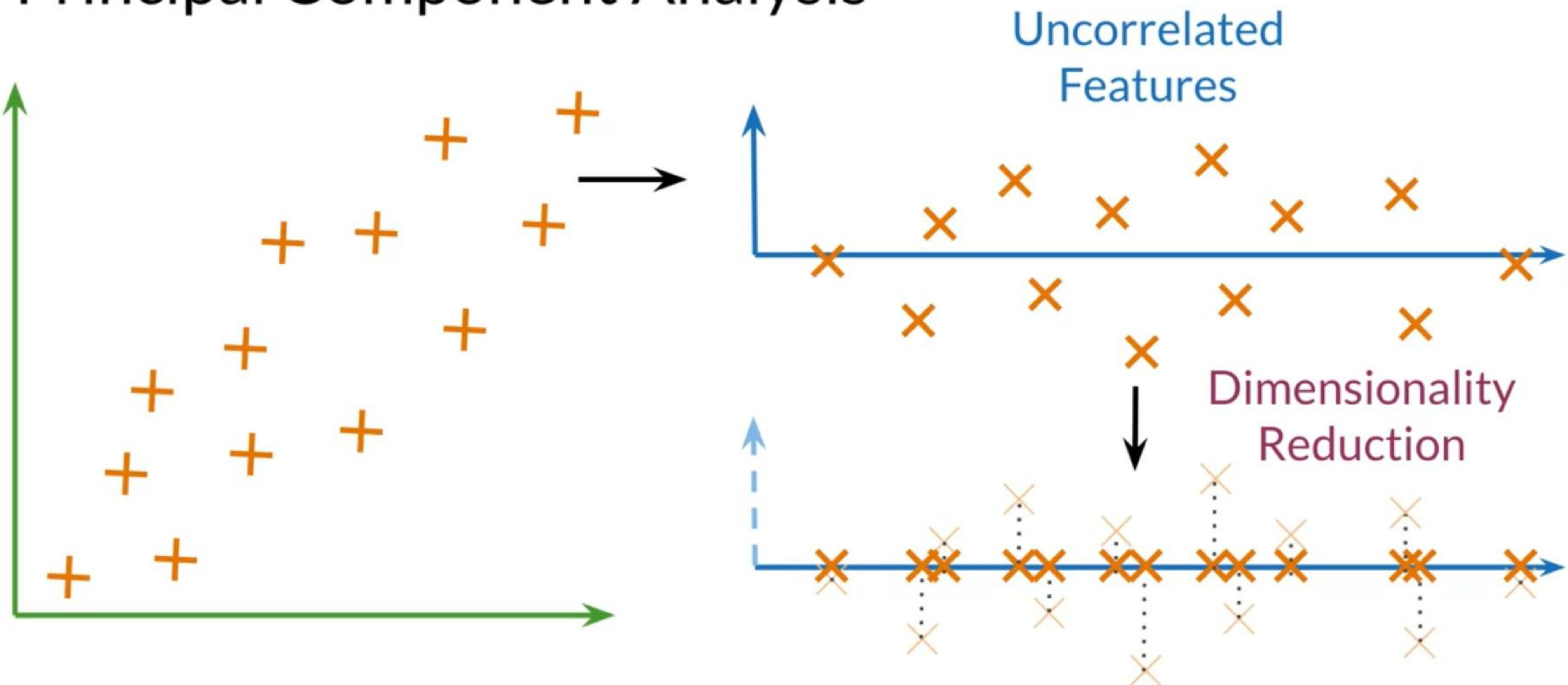
Summary

- Original Space $\longrightarrow$ Uncorrelated features $\longrightarrow$ Dimension reduction
- Visualization to see words relationships in the vector space

Outline

- How to get uncorrelated features
- How to reduce dimensions while retaining as much information as possible

Principal Component Analysis

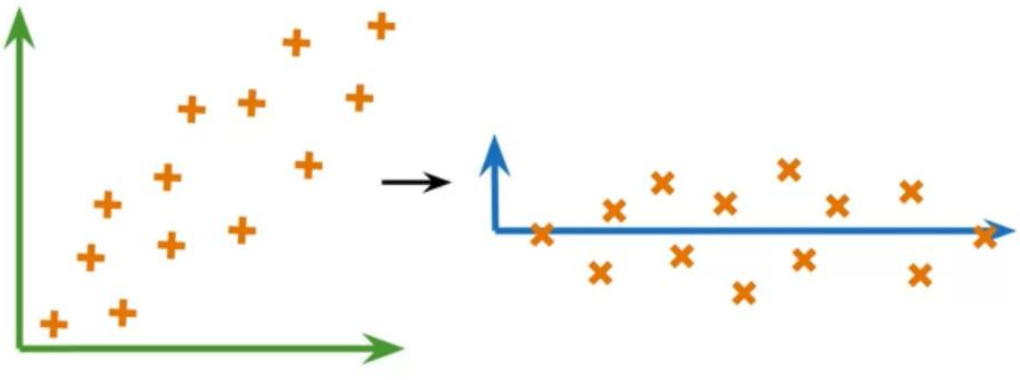


PCA algorithm

Eigenvector: Uncorrelated features for your data

Eigenvalue: the amount of information retained by each feature

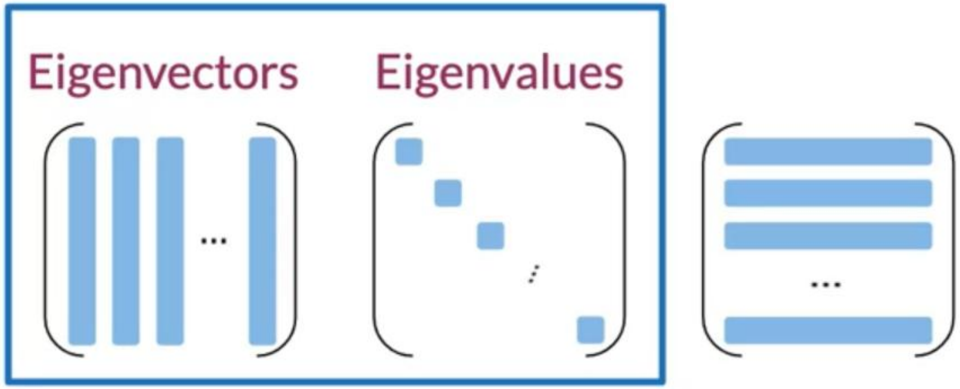
PCA algorithm



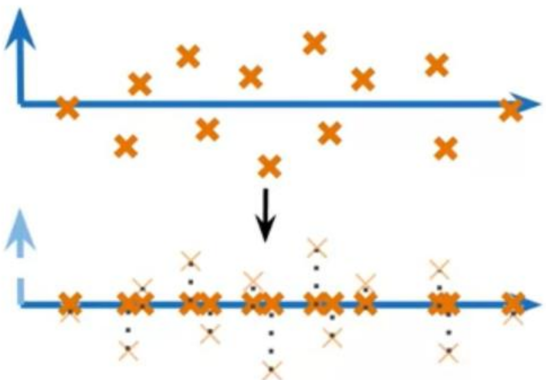
Mean Normalize Data $$x_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}}$$

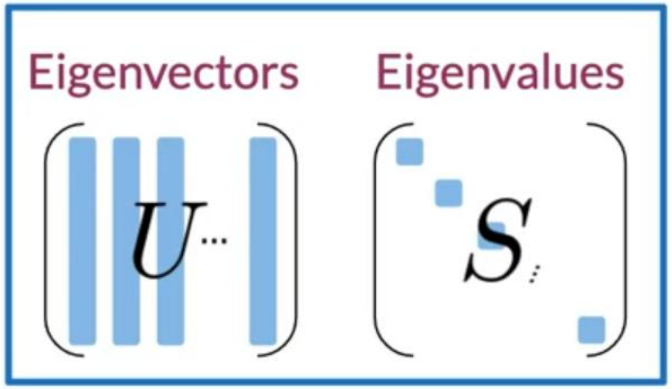
Get Covariance Matrix Σ

Perform SVD $\mathrm{SVD}(\Sigma)$



PCA algorithm





Dot Product to Project Data $X' = XU[:, 0:2]$

Percentage of Retained Variance $\dfrac{\sum_{i=0}^{1} S_{ii}}{\sum_{j=0}^{d} S_{jj}}$

Summary

- Eigenvectors give the direction of uncorrelated features
- Eigenvalues are the variance of the new features
- Dot product gives the projection on uncorrelated features