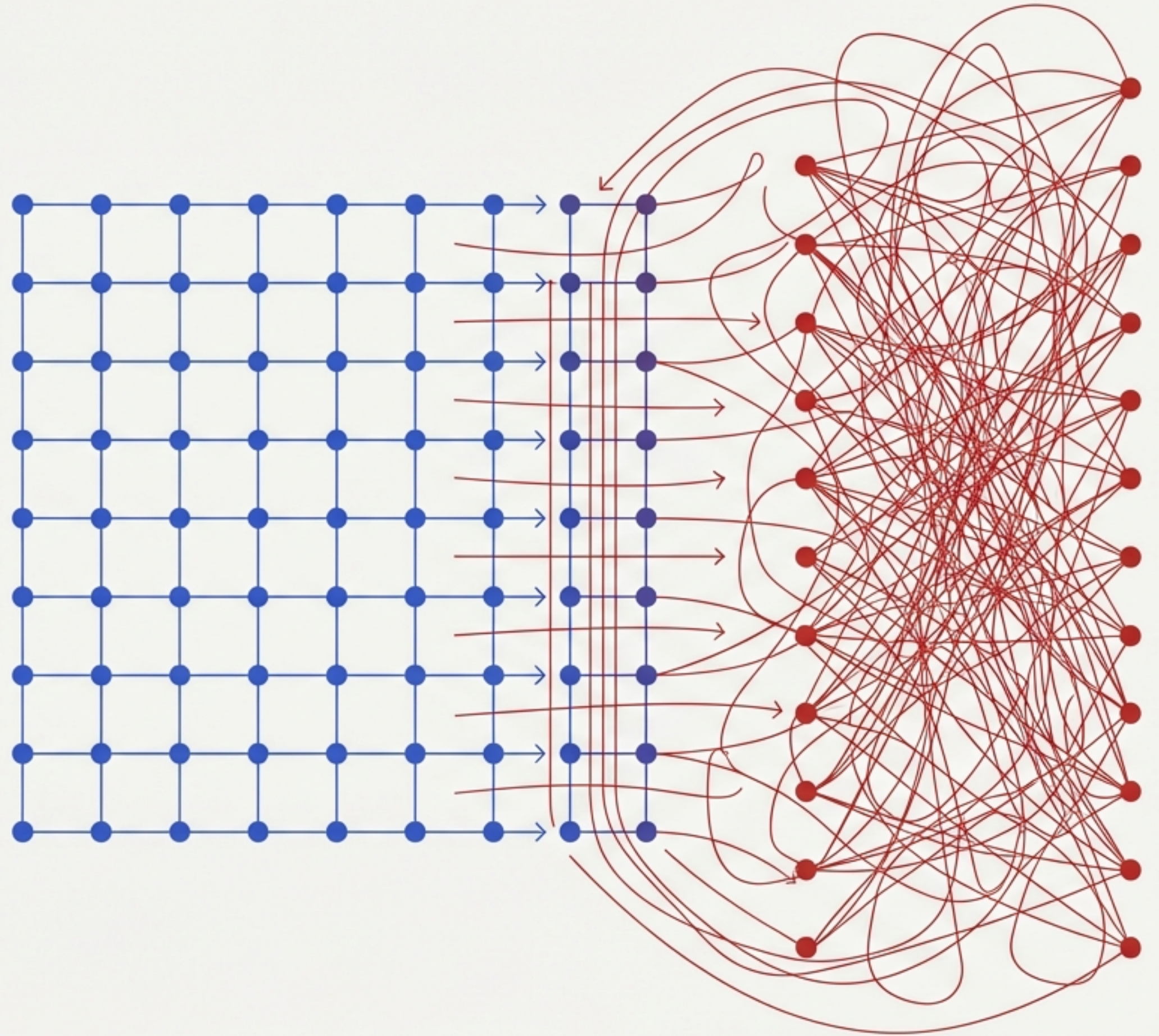


Algebraic Models for Social Networks

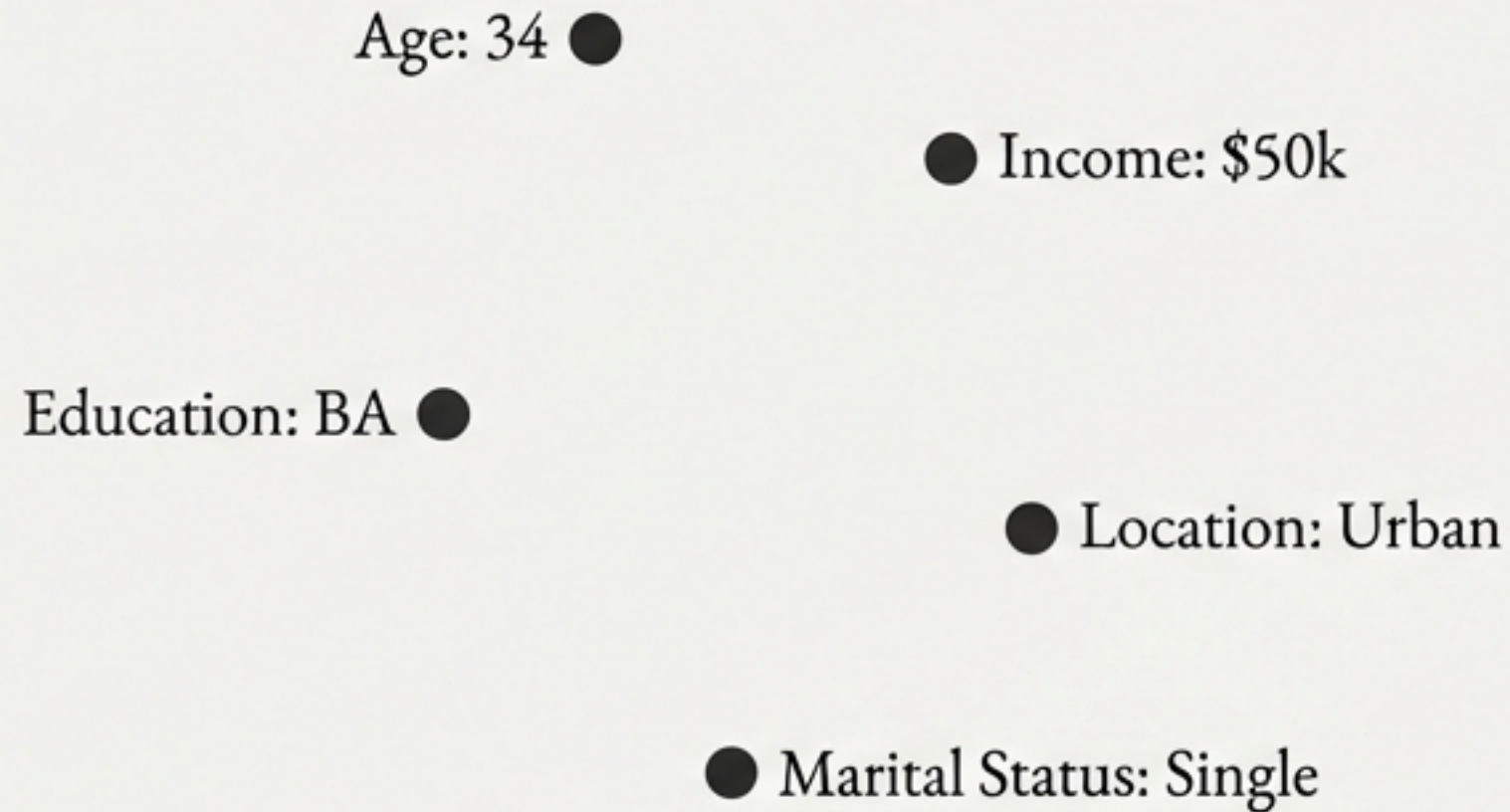
Uncovering the Structural Logic of Human Relations



Synthesized from the work of Philippa Pattison

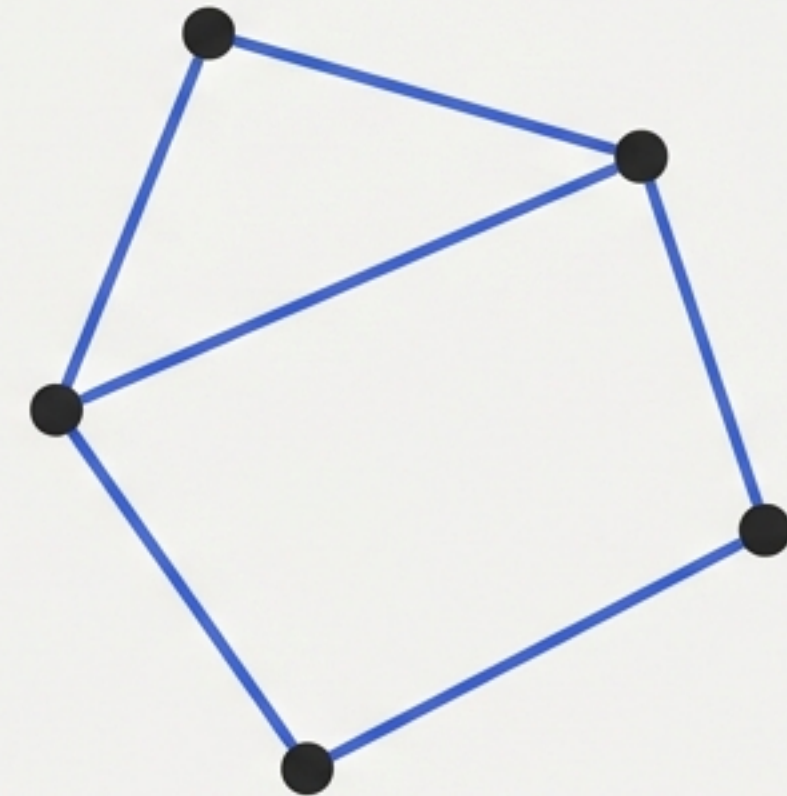
Moving Beyond the Variable

Attribute-Based Sociology (Reductionism)



Focuses on individual attributes. Explains behavior via isolated characteristics.

Structural Analysis

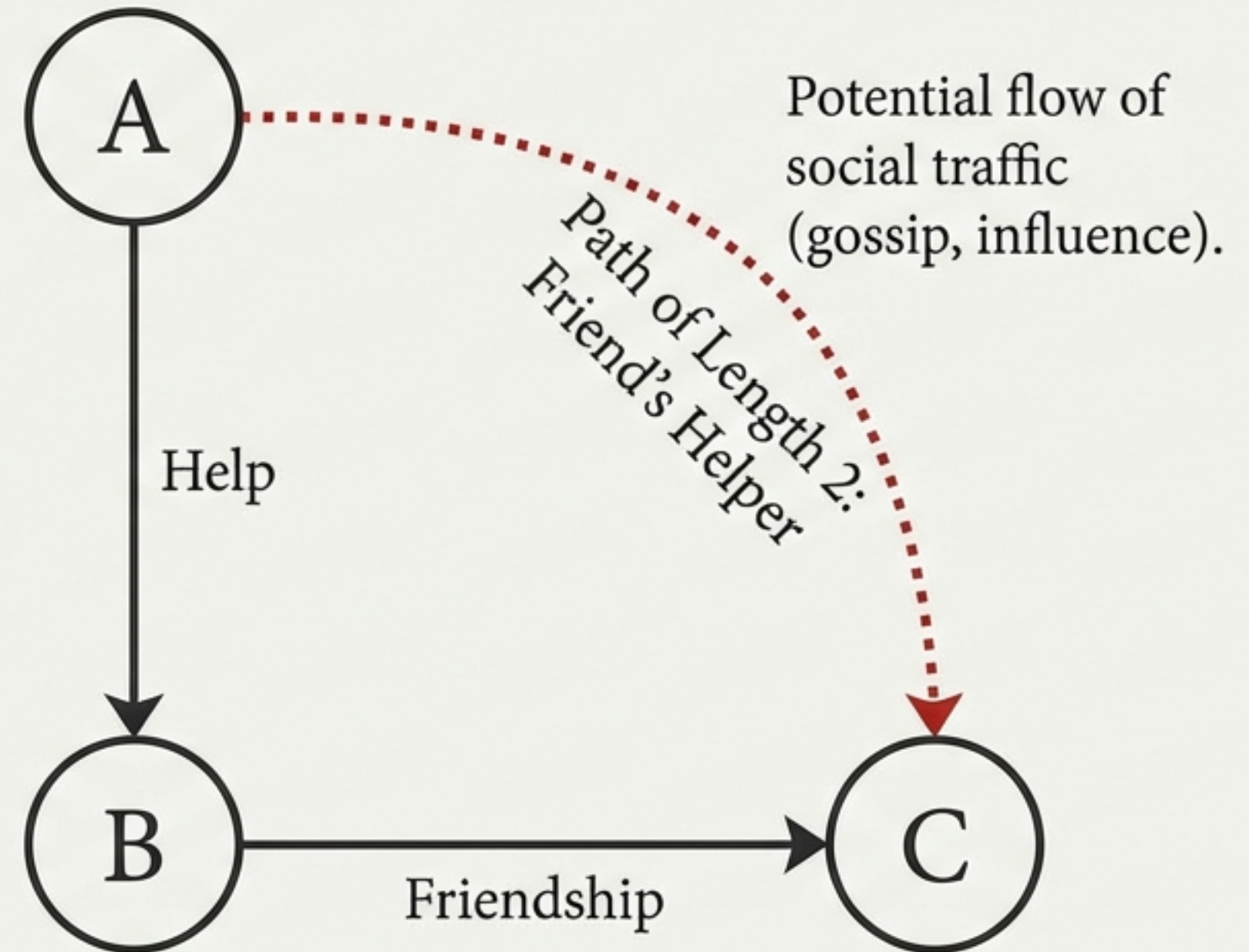


Focuses on relations among entities. Explains behavior via the context of connections.

“The structural perspective stresses the importance of relations rather than the atomisation of reductionism.”

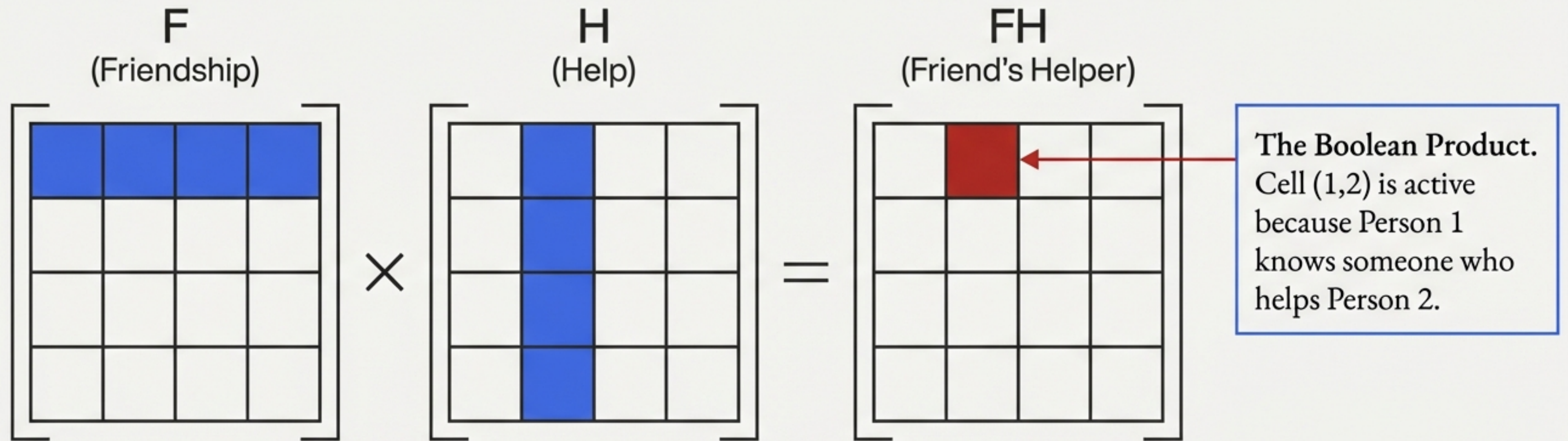
Networks are Pathway for Social Traffic

A network is not a static map; it is a conduit for the flow of information, influence, and contagion. To understand this flow, we must analyze Compound Relations (paths).



The Algebra of Interlock

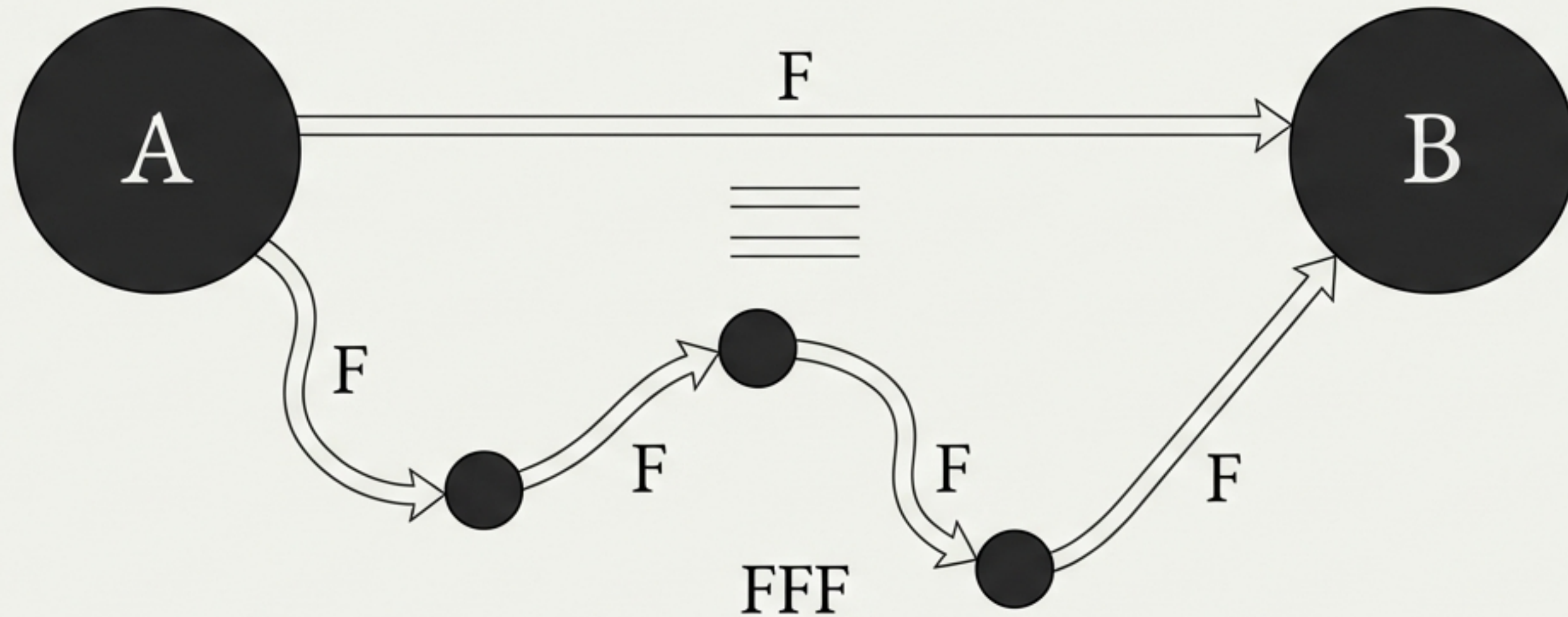
Relational Composition via Boolean Product



Logic: If A is related to B by F, and B is related to C by H, then A is related to C by FH.

The Axiom of Quality

Social networks allow for infinite paths ($Friend \rightarrow Friend \rightarrow Friend \dots$). Structural analysis becomes possible only when we equate long paths with shorter ones, reducing infinite complexity to finite structure.



The Code Breaker: When 'My friend's friend's friend' is structurally equivalent to 'My friend' ($\$FFF = F\$$).

The Partially Ordered Semigroup

The Multiplication Table of Society

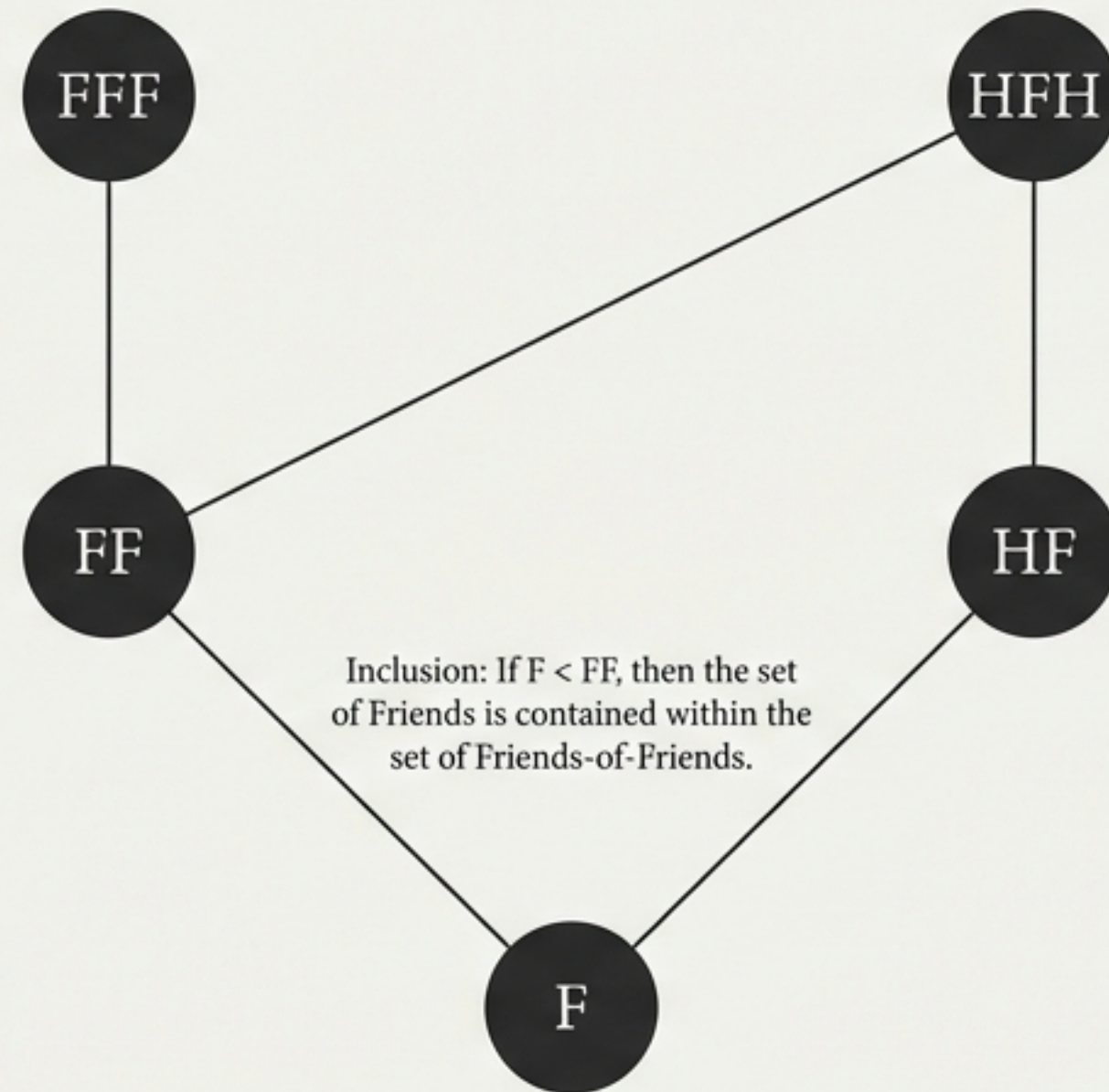
	F	H	FF	FH	HF	HH	FFF	HFH
F	FF	FH	FH	HF	FHF	FFF	F	HFF
H	HF	HFF	HFH	HHF	HHH	H	H	H
FF	FFF	FHF	FFFF	HFH	HHH	HHHF	HFHF	HFHH
FH	FFH	FHF	HFH	FFFFF	FHFH	HFFFF	HHFF	HHHH
HF	HHH	HHHFH	HHHFH	HFFHF	HFHFH	HFHHH	HHHFF	HHHHH
HH	FFFFFF	FHFHFH	HFFFFFF	HHFFFFFF	HHFFFFFF	HFFHHH	HHHHIH	HHFHFH
FFF	FFFFFFH	HFFFFFF	HHFFFFFF	HHFFFFFF	HFHHHF	HHHHIH	F	FFFFFFH
HFH	FFFFHH	FHFHH	HHHHF	HFHFH	HHHHF	HHHHH	FFFFFFF	FHFHFH

This table represents the “Code of the Group.”

It maps every possible combination of relationships into a finite set of distinct structural types.

The Hierarchy of Relations

Visualizing Inclusion via the Hasse Diagram

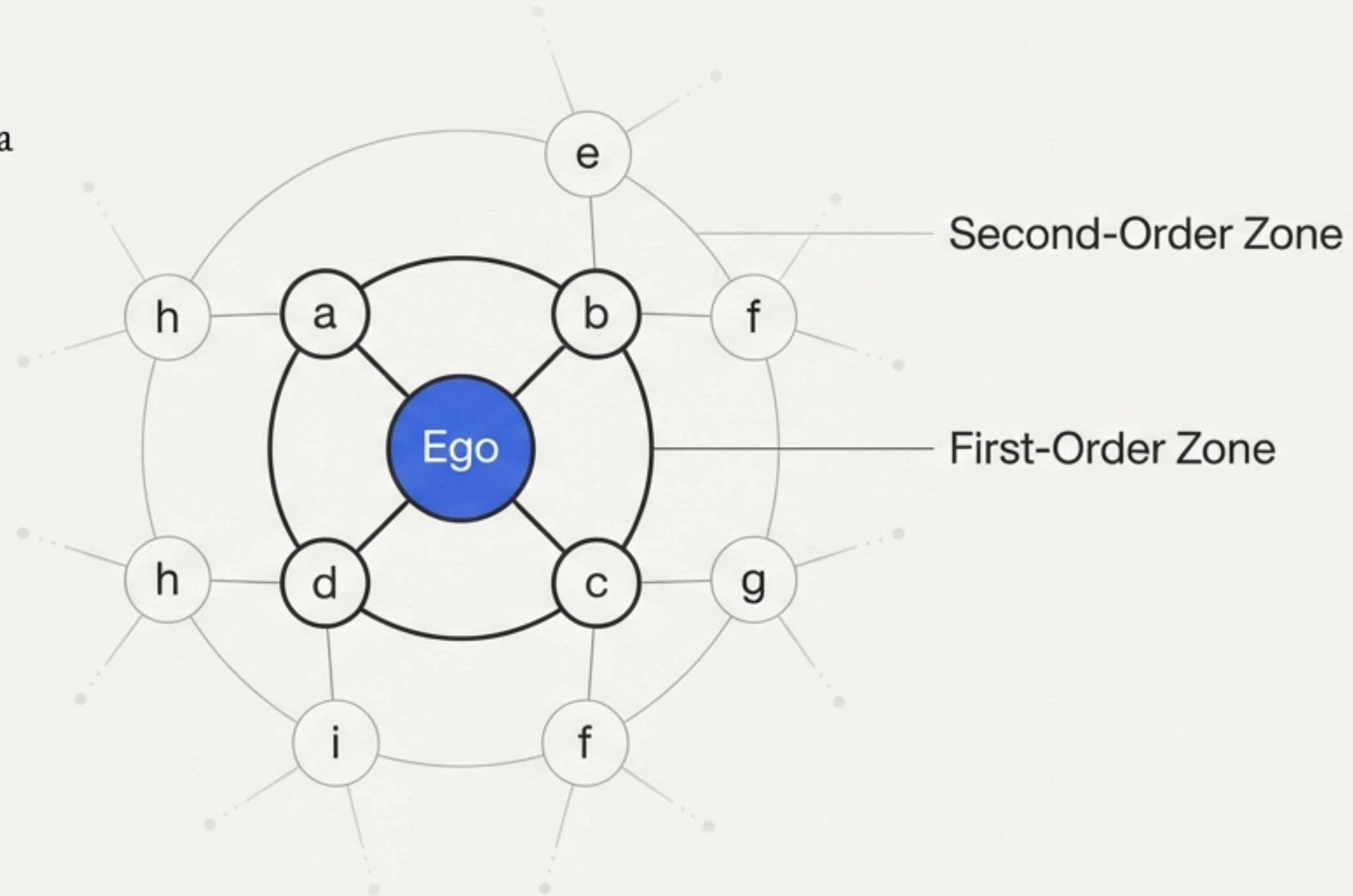


This diagram demonstrates the “deep structure” of the group—showing which relations imply others.

The Ego-Centric View

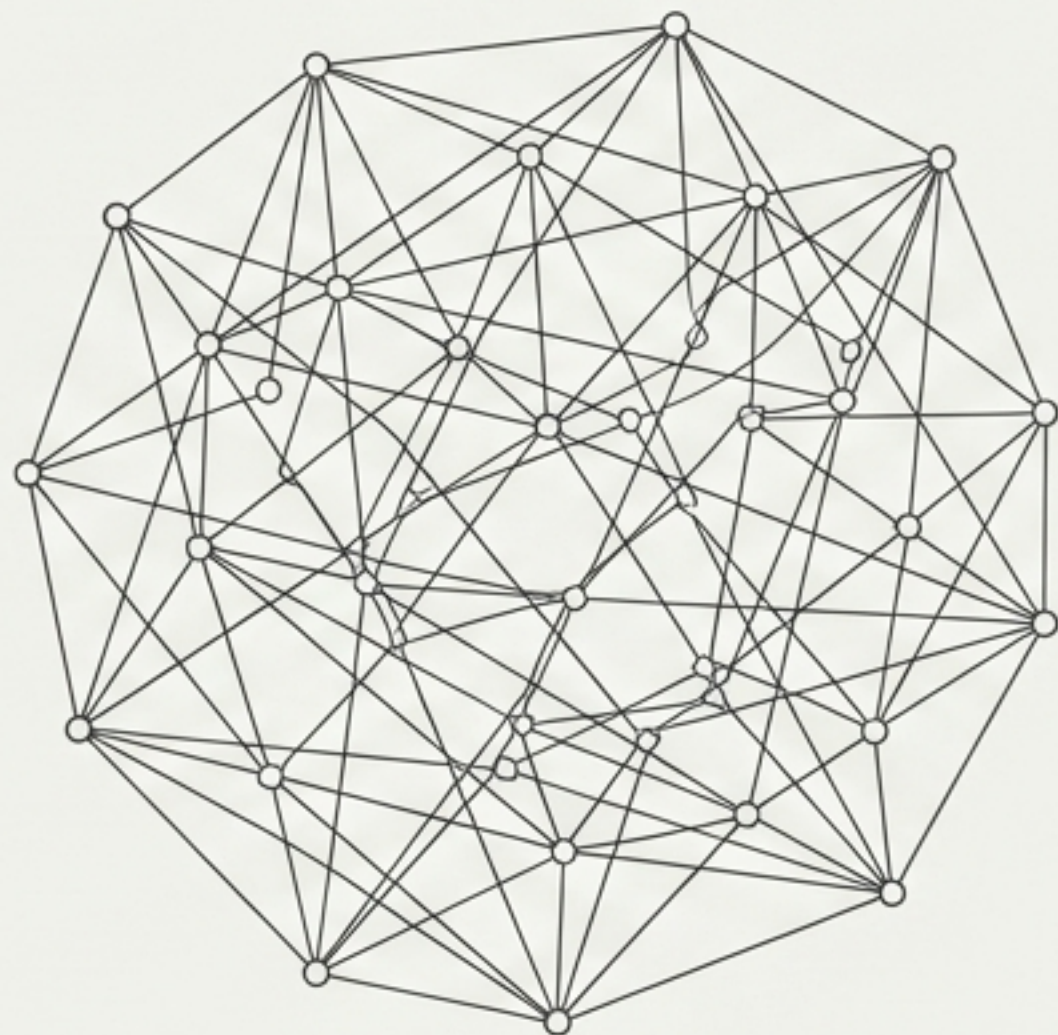
The Problem of the Horizon

We rarely possess the complete map.
We must model structure based on what one person (Ego) can see, up to a truncated horizon.

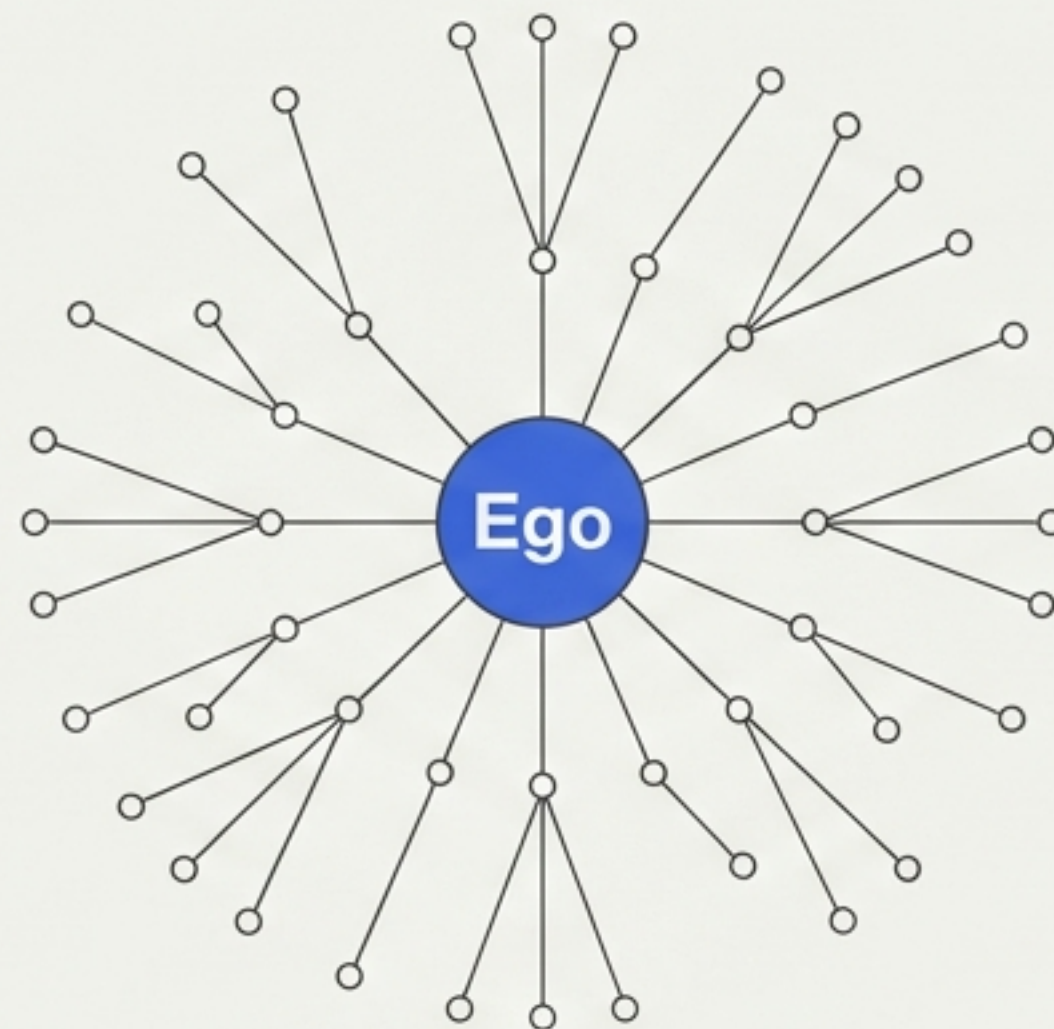


Local Role Algebras

Instead of a global multiplication table, we build an algebra based on paths emanating from Ego. We generate paths (1^*A , 1^*AA) and compare the subsets of people they reach.



Complete Network
(Global Semigroup)



Local Role Algebra (Q_1)

If Path A and Path B reach the exact same set of people, then $A = B$ in Ego's algebra.

Modeling the Horizon

Partial Role Algebras

Observable Structure ($k = 2$)

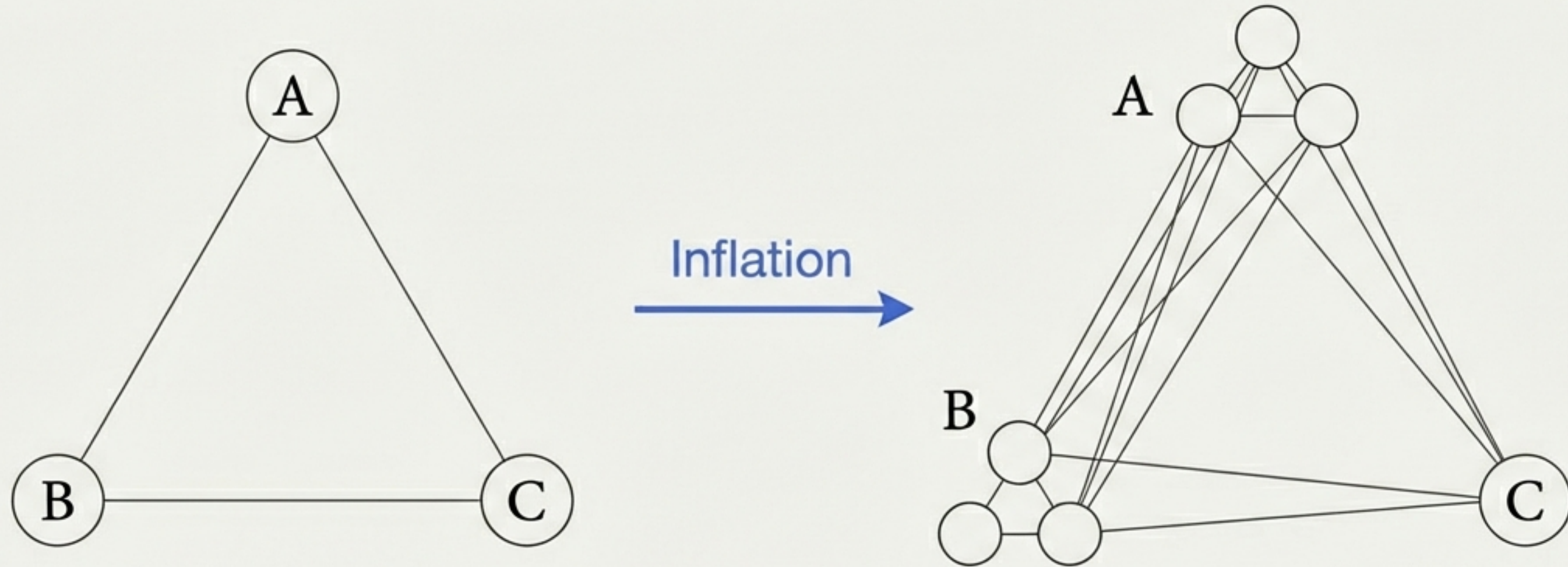
Truncated
Relation Plane

	Length 1					Length 2					Length 3						
Ego	0	1	1	1	0	1	1	1	0	1	0	1	1	0	1	0	1
a	1	0	1	0	0	1	1	0	1	0	1	0	1	0	0	1	0
b	1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
c	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
d	0	0	0	1	0	1	0	1	0	1	0	0	1	0	0	1	0

We construct a Partial Role Algebra based only on paths of length k . This allows rigorous modeling of **snowball samples** without assuming knowledge of the infinite world.

When Are Two Networks Identical?

Isomorphism and Structural Equivalence



$$\text{Semigroup}(N_{small}) \cong \text{Semigroup}(N_{large})$$

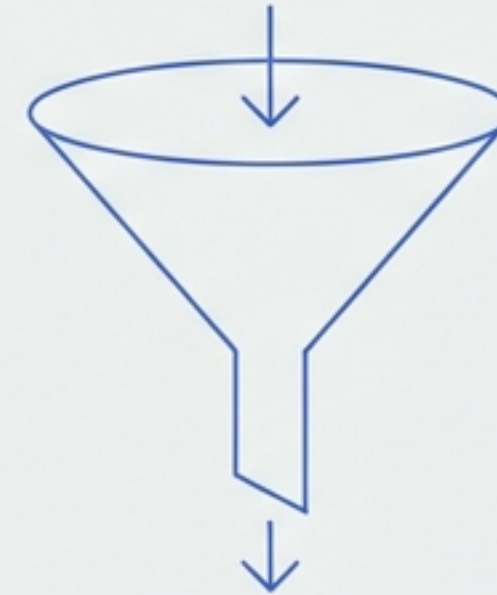
If we replace an individual with a group of clones (structurally equivalent people), the algebraic logic of the network remains unchanged.

Mapping Complexity to Simplicity

Homomorphism and Blockmodeling

1	0	0	1	0	1	1	0	0	0
0	1	0	1	1	0	0	1	1	1
0	1	1	1	0	1	1	0	0	0
0	0	1	1	0	0	0	1	0	1
0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	1	0	1	1	1
1	0	1	0	0	1	1	0	1	0
0	0	0	0	1	1	0	1	1	1
1	0	1	1	0	0	0	0	1	0
0	0	0	0	1	1	1	0	1	1

Homomorphism

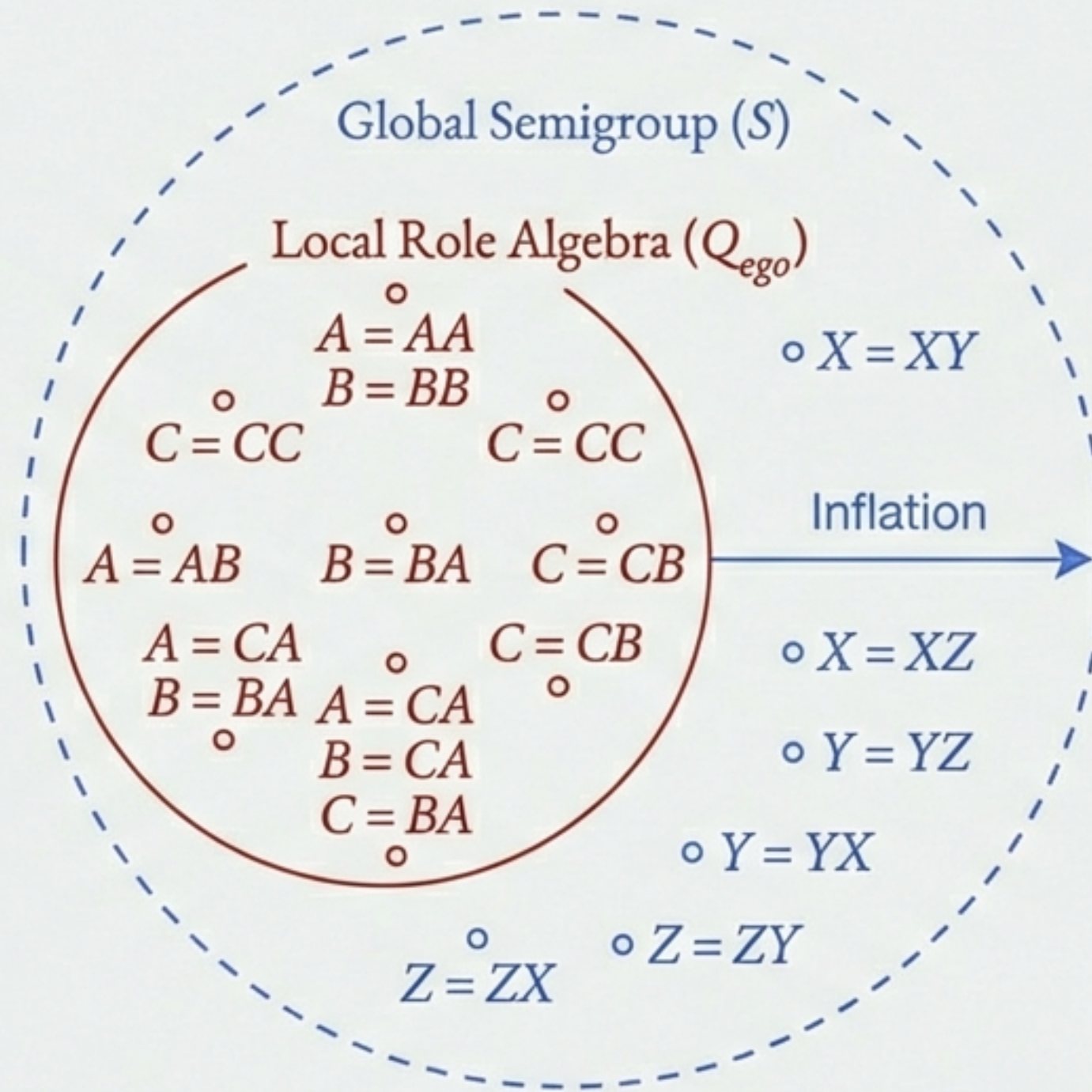


1	0
1	1

The Image Matrix:
A structural skeleton
of the complex reality.

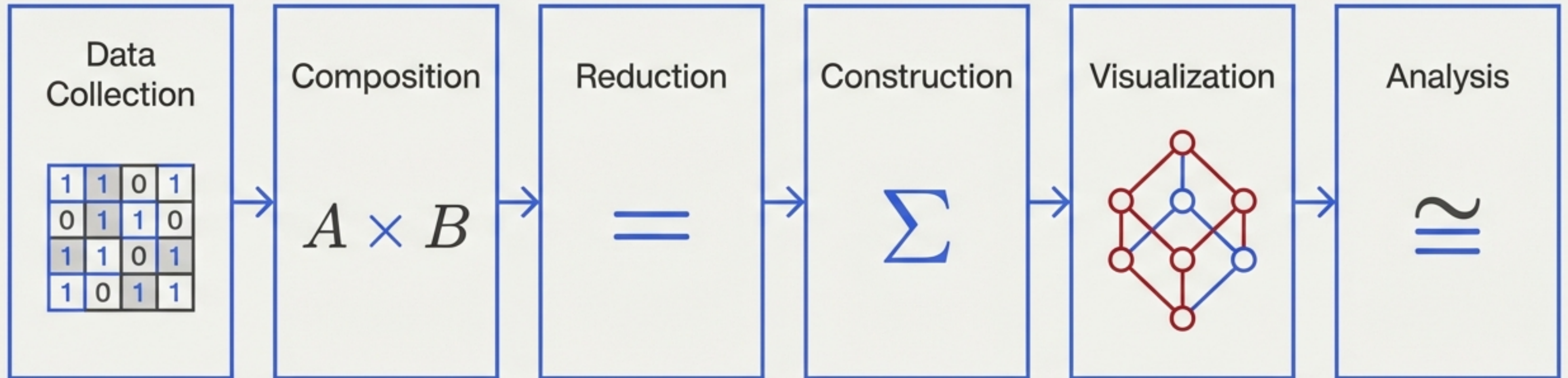
We partition actors into 'Blocks' (Positions) to reveal the underlying logic. The complex network maps to a simpler semigroup that preserves the essential orderings.

The Local is Nested in the Global



The algebra of the entire network is the master structure. All local (Ego) algebras are specific instances nested within it. Any distinction made locally is valid globally.

The Analytical Workflow



From raw binary matrices to algebraic insight, this workflow converts social context into a computable object.

From Context to Computation

Context \rightarrow *Structure* \rightarrow *Algebra*

Social context is no longer a vague background variable. Using algebraic models, it becomes a rigorous, computable object with distinct generative logic.